

# Teoria do Consumidor: Demanda

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## 1 Demanda e Renda

- Curvas de renda-consumo e de Engel
- Elasticidade Renda
- Ilustrações

## 2 Demanda e Preço

- Curvas preço-consumo e curva de demanda
- Elasticidade preço
- Bens de Giffen
- Elasticidade preço cruzada
- Relações entre as elasticidades

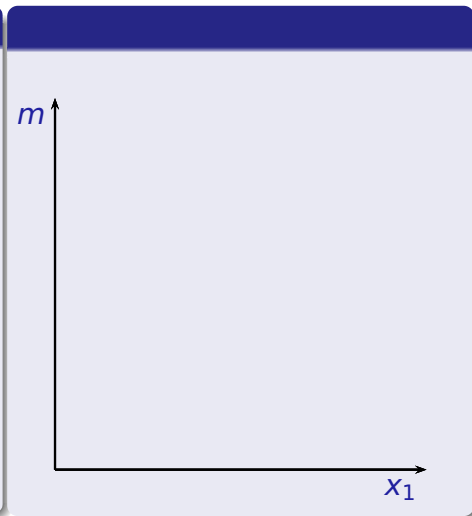
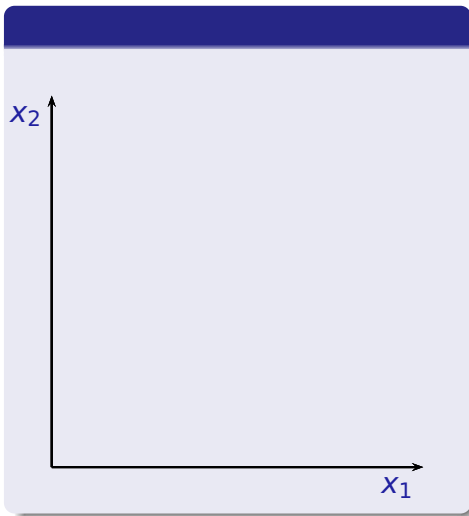
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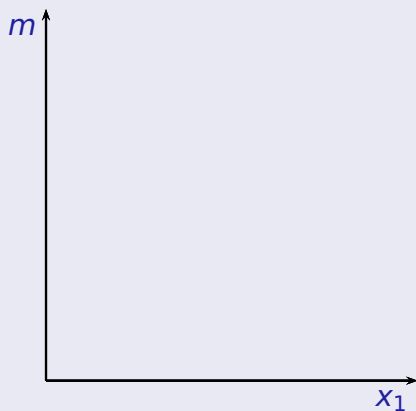
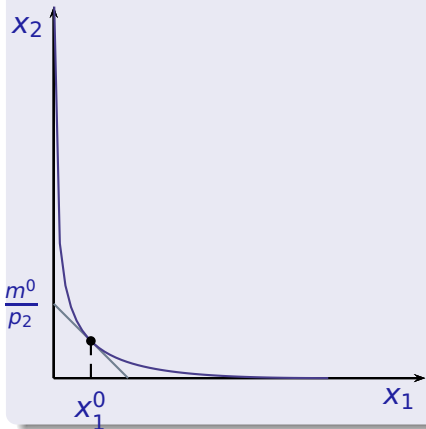
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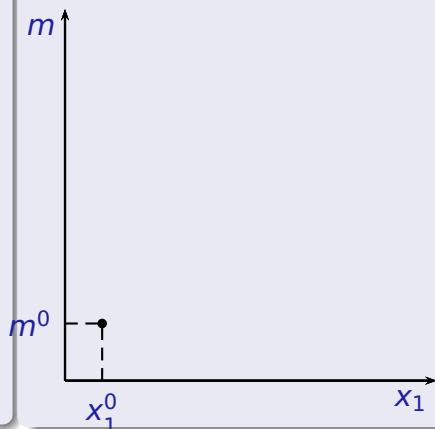
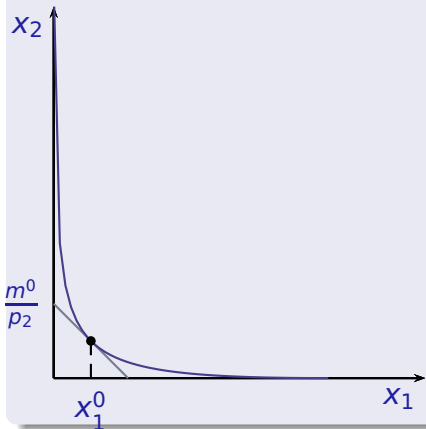
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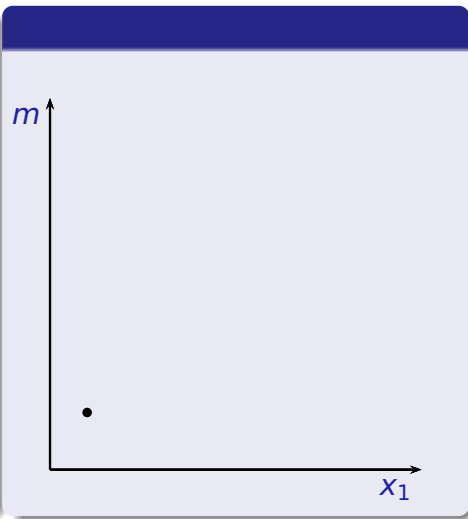
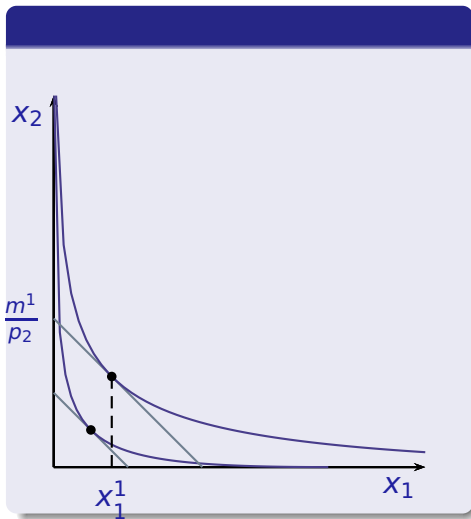
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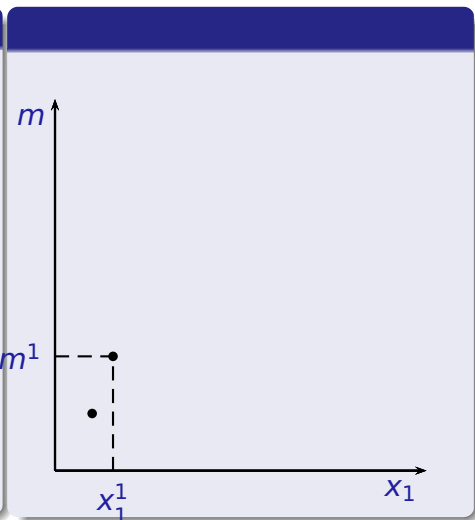
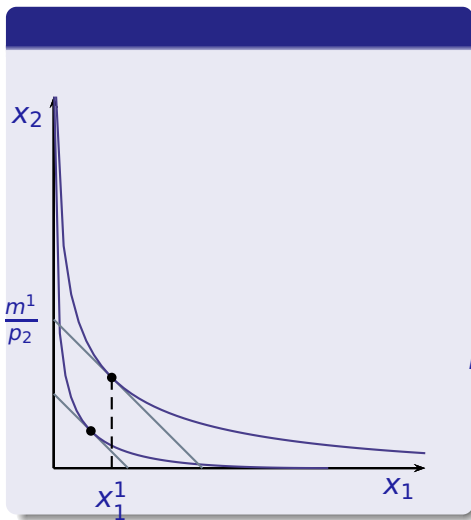
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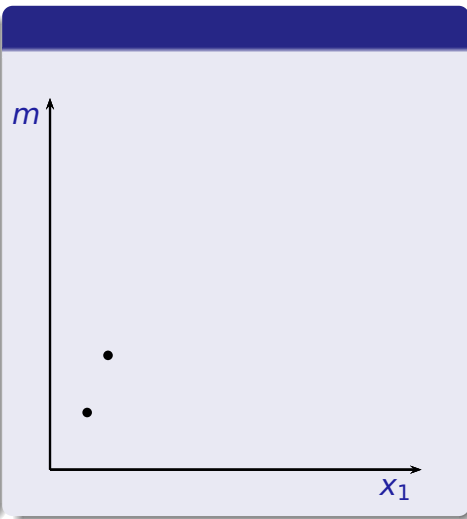
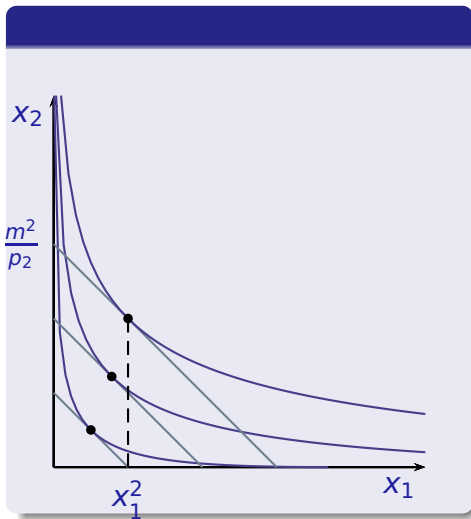


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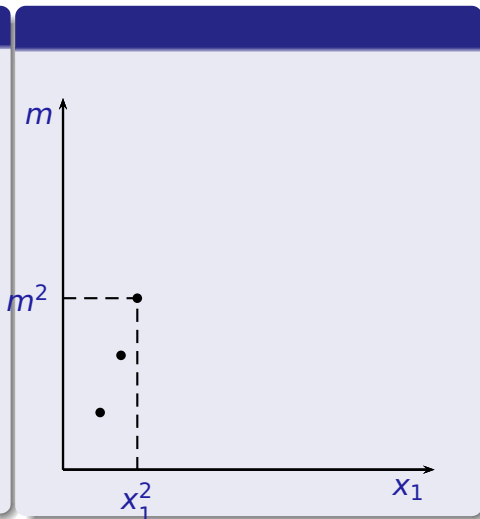
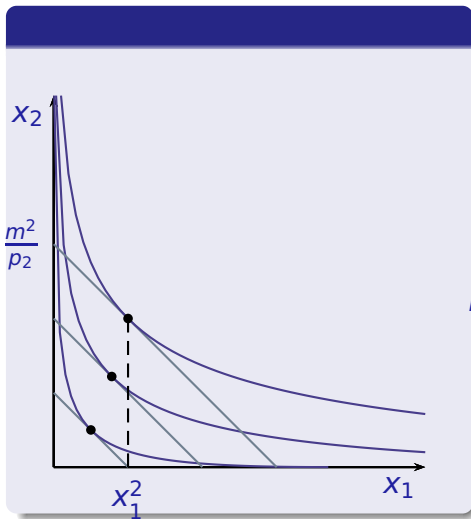




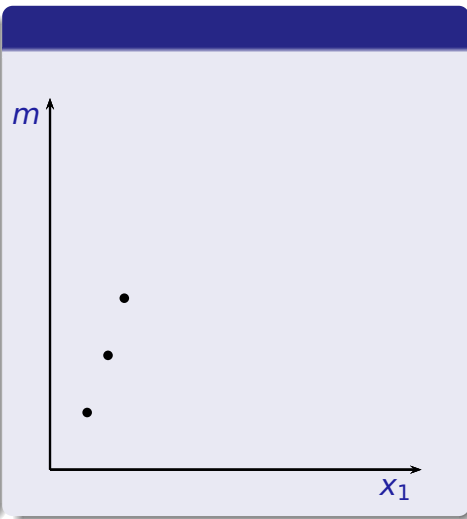
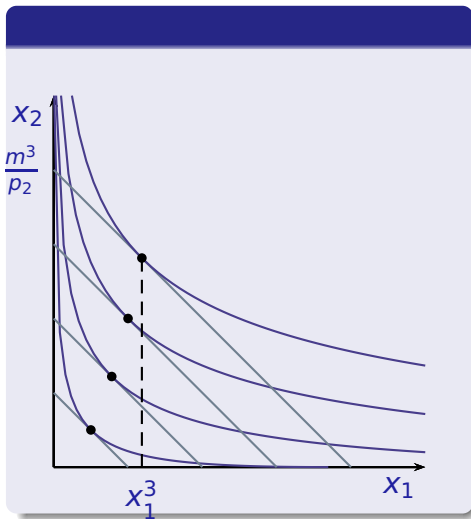
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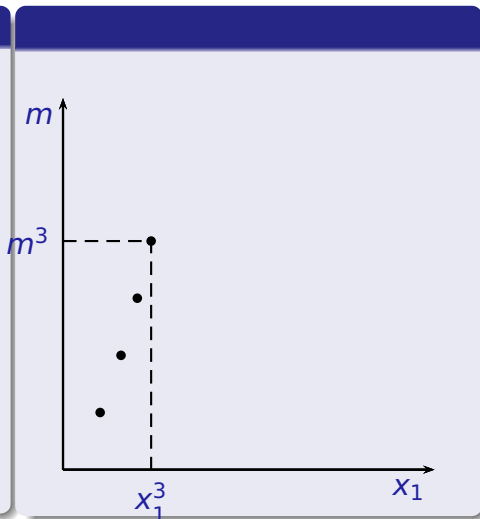
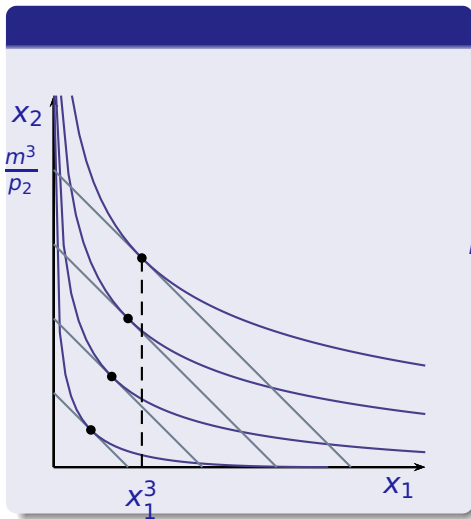
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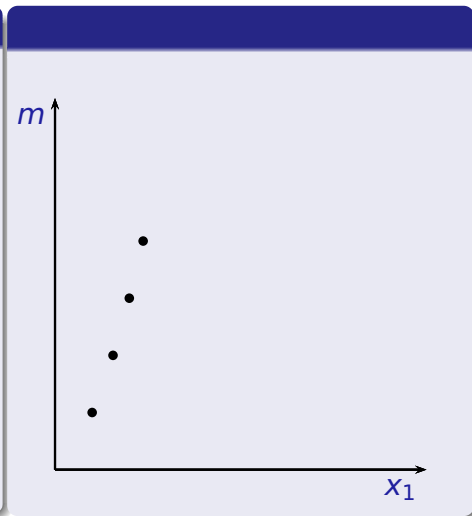
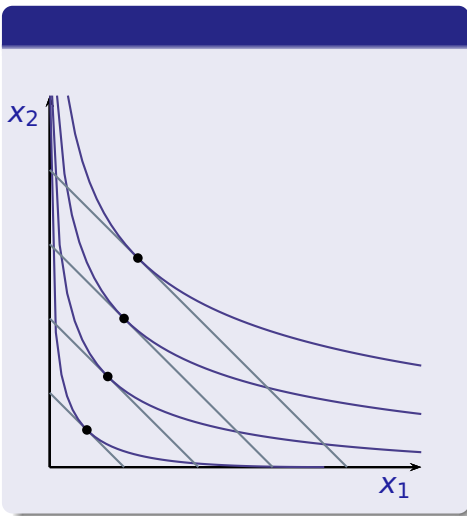
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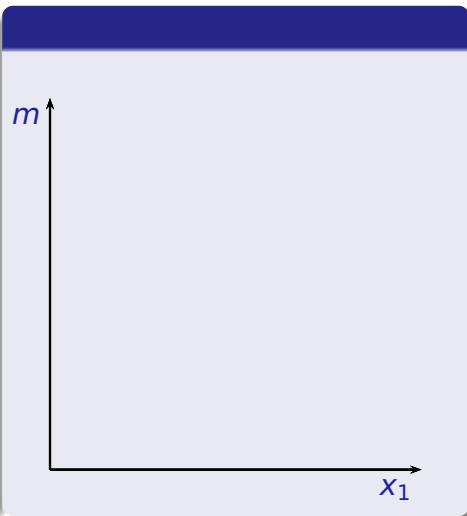
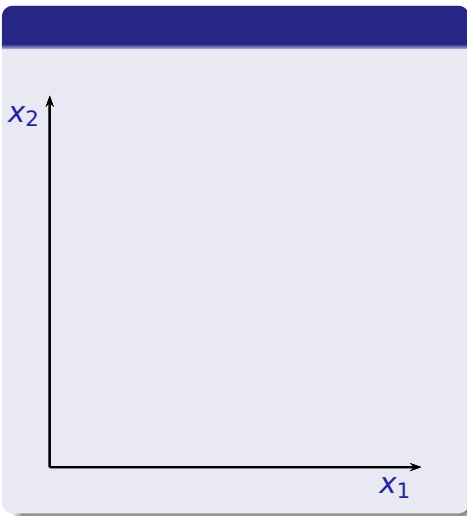
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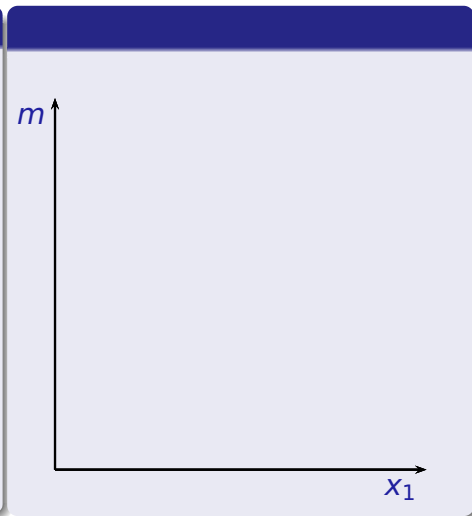
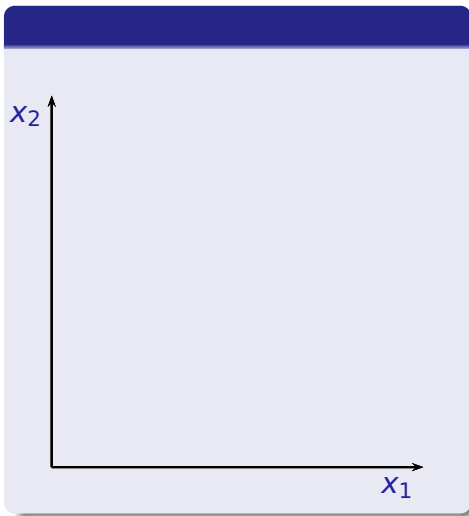
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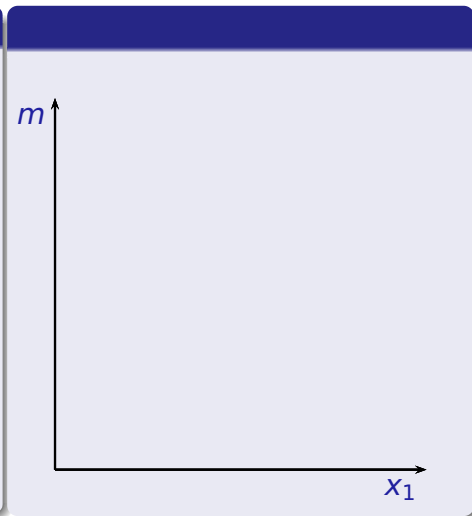
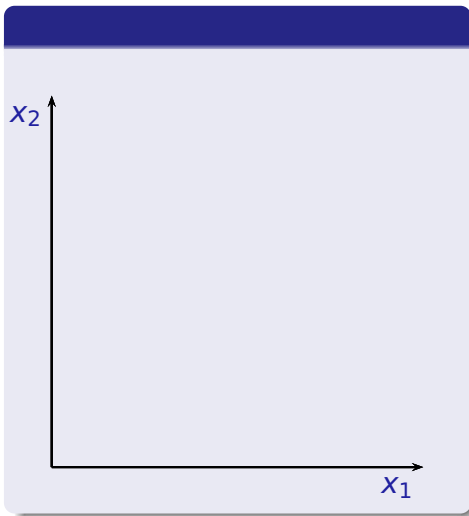
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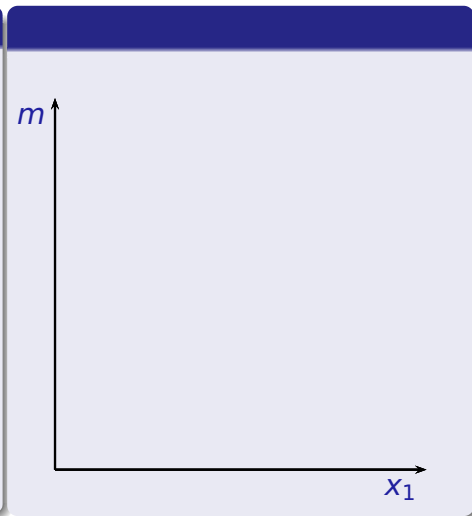
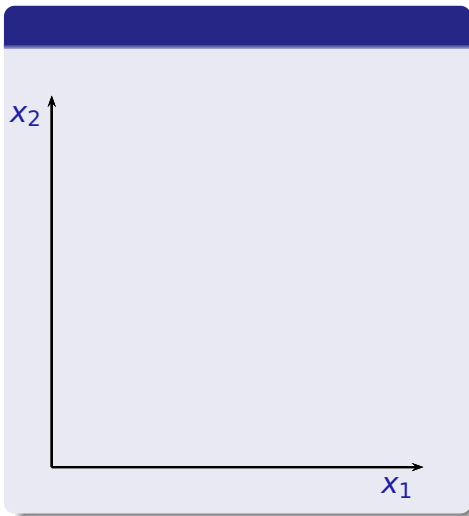


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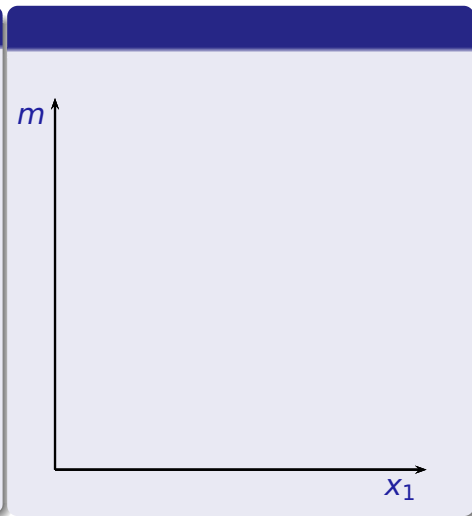
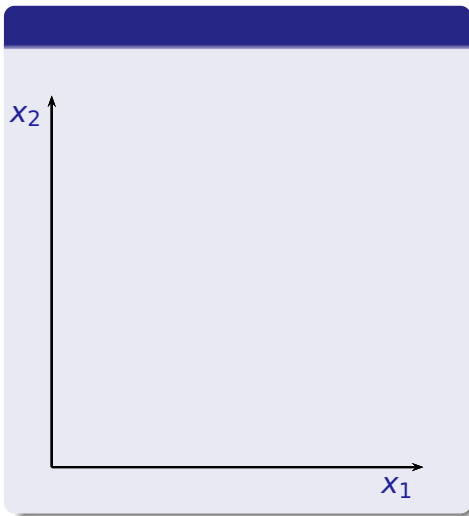




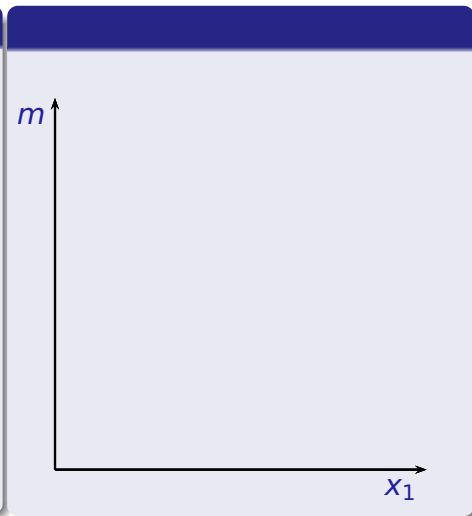
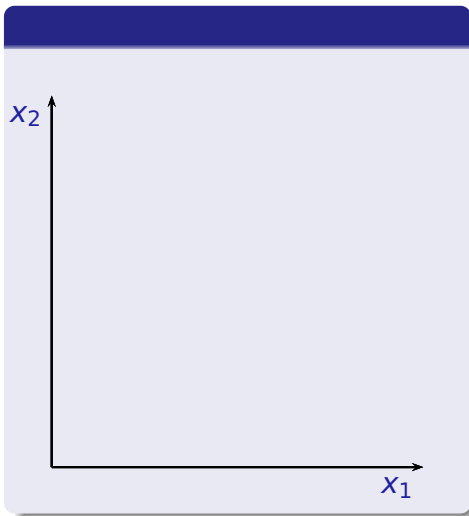
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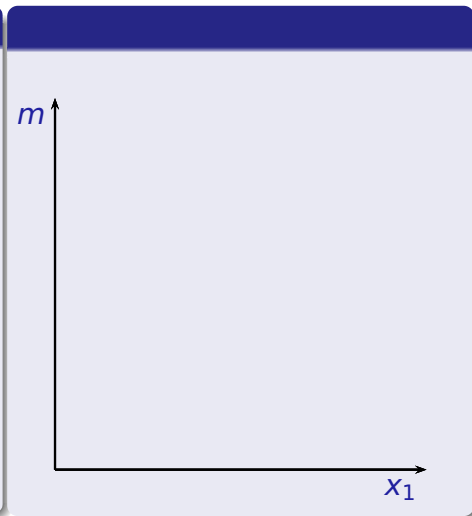
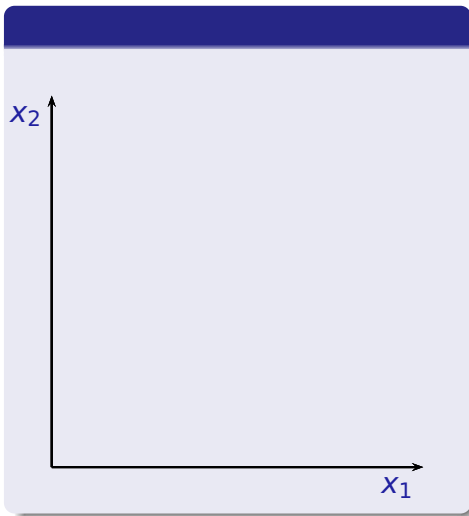
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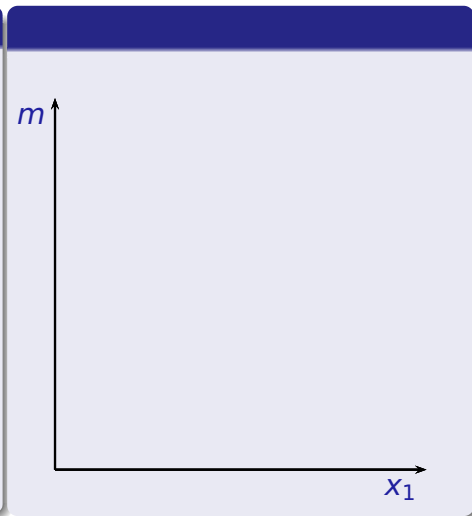
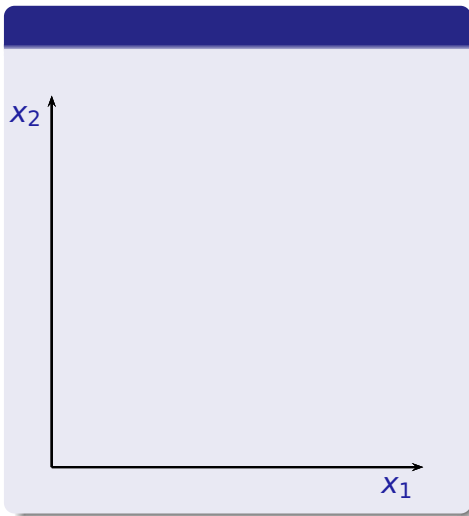
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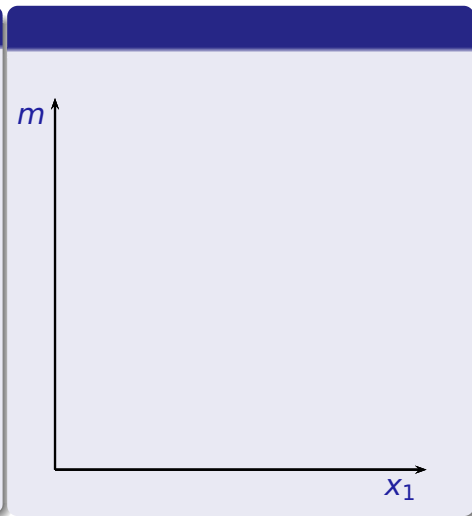
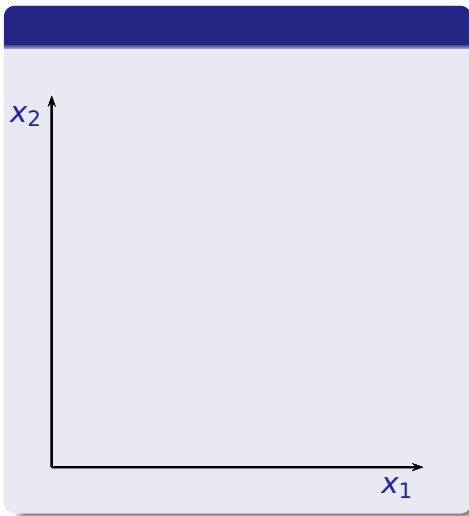
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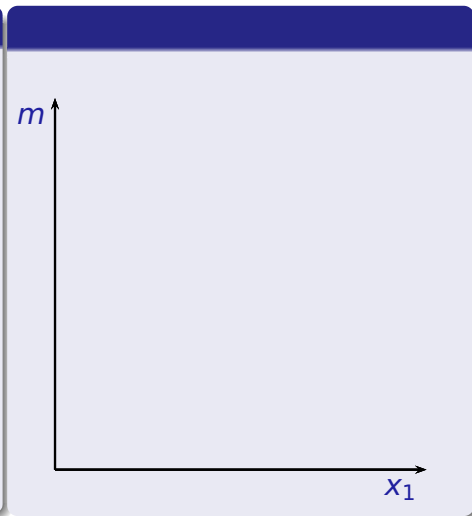
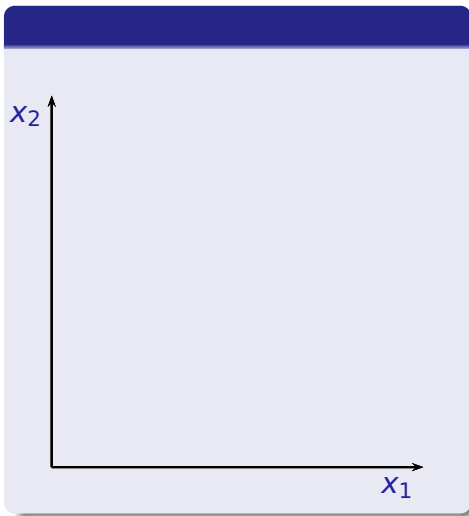
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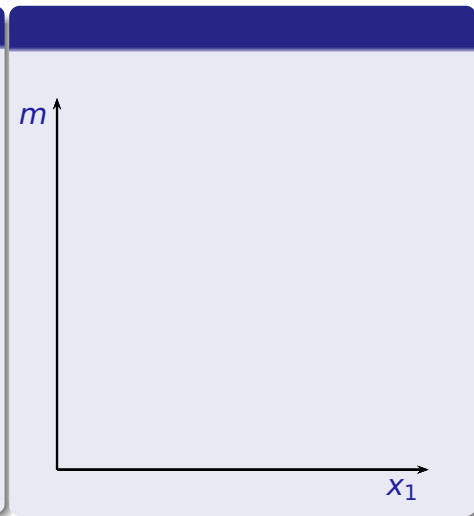
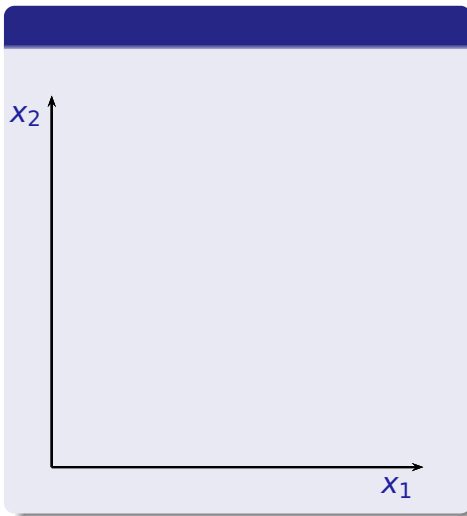
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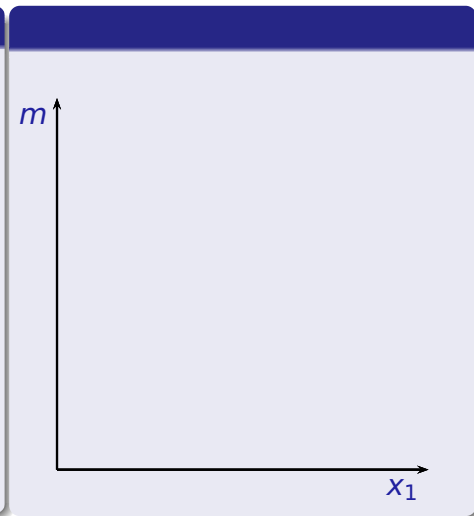
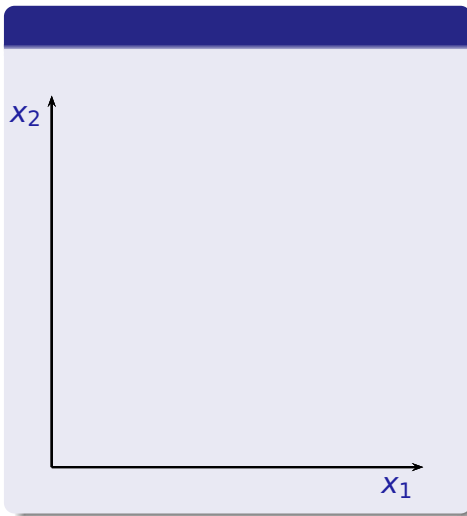


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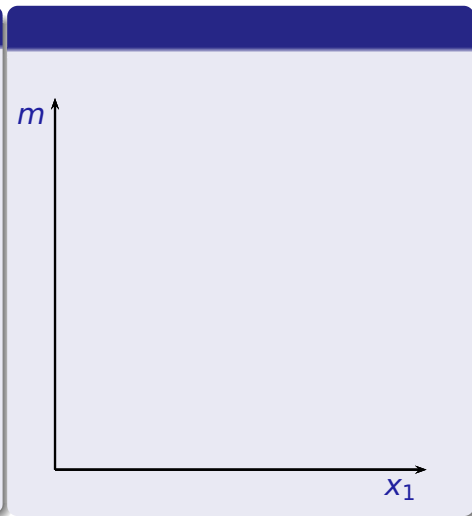
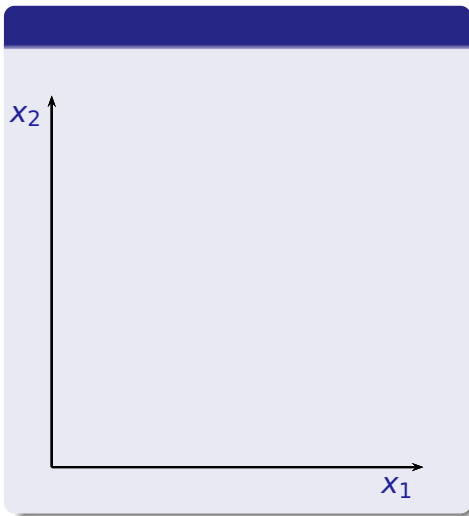




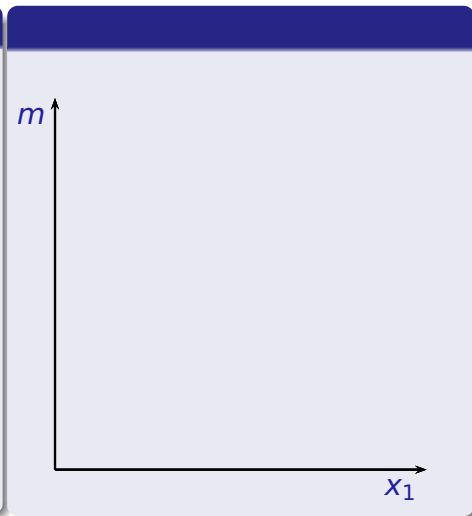
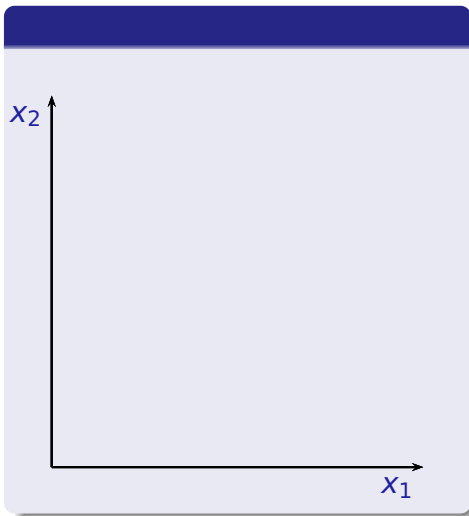
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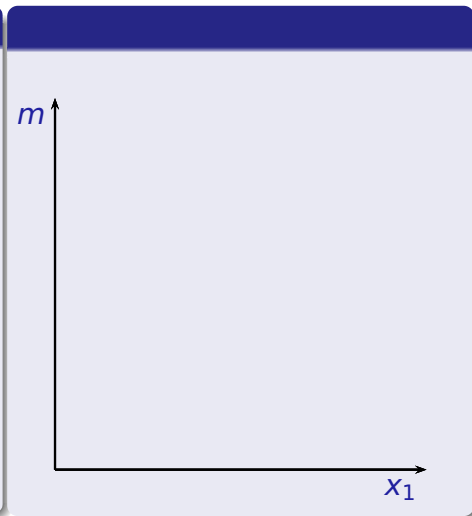
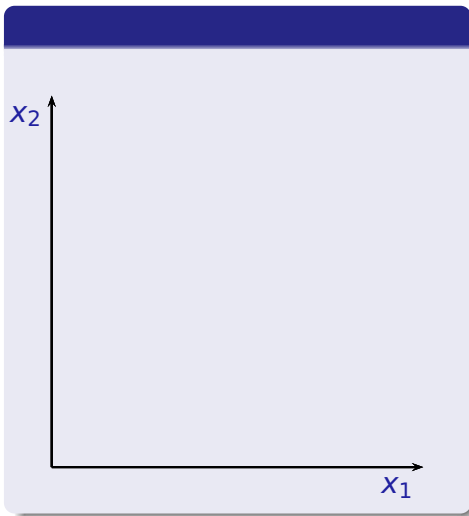
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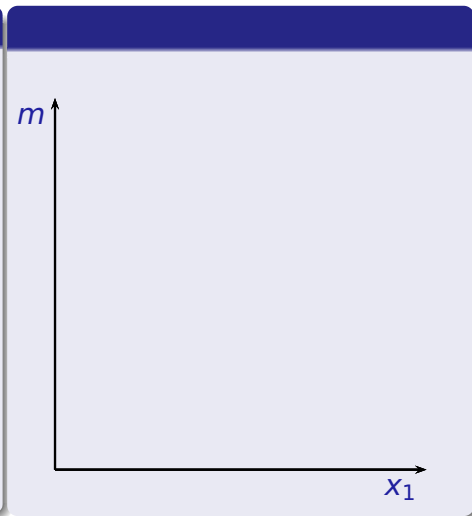
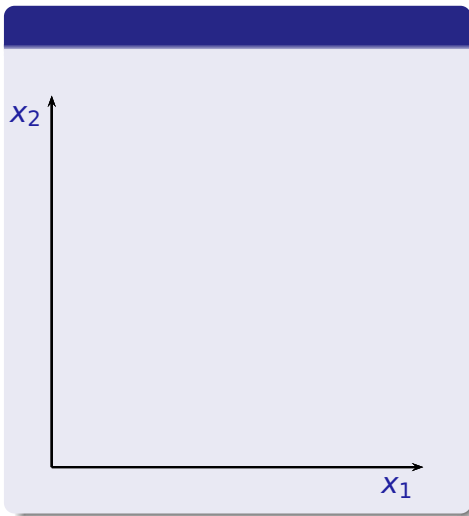
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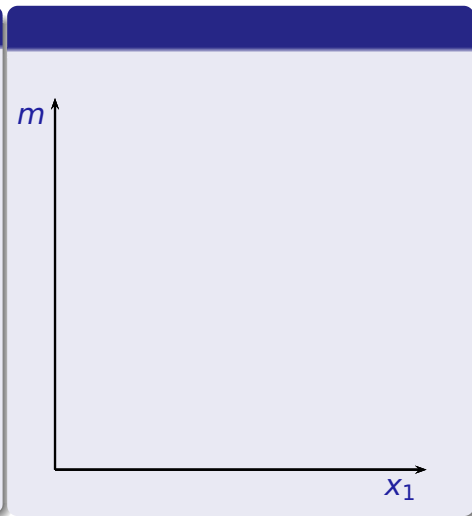
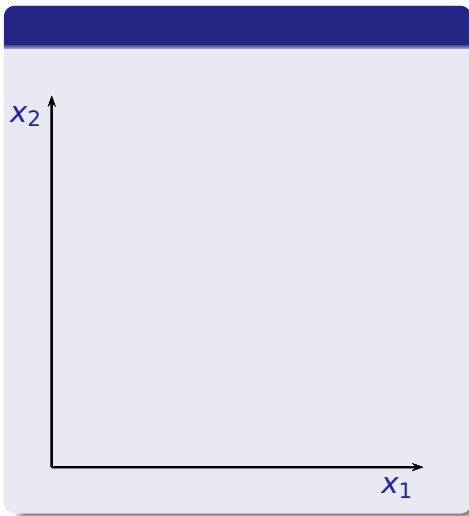
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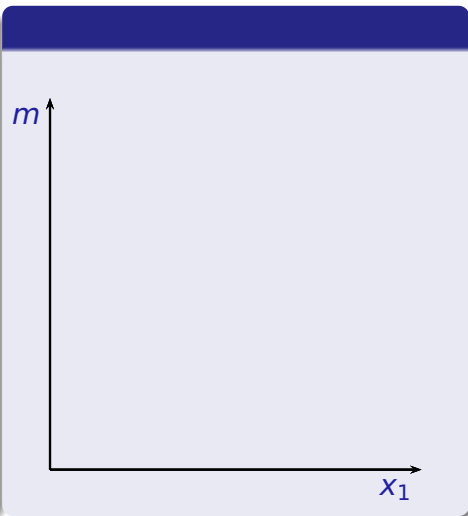
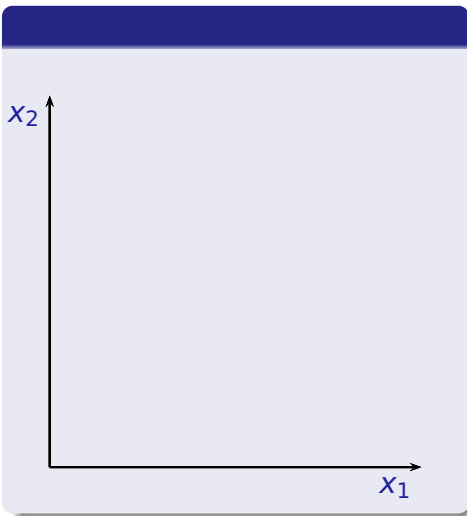
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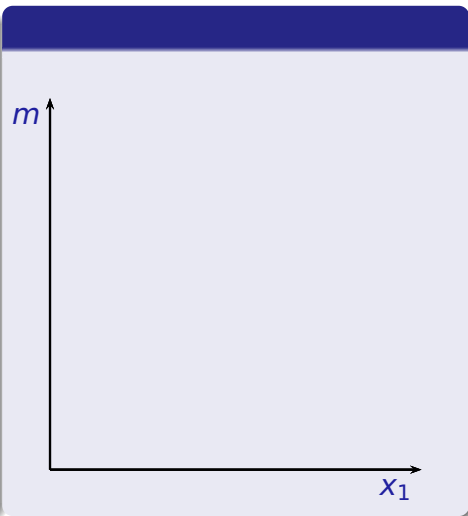
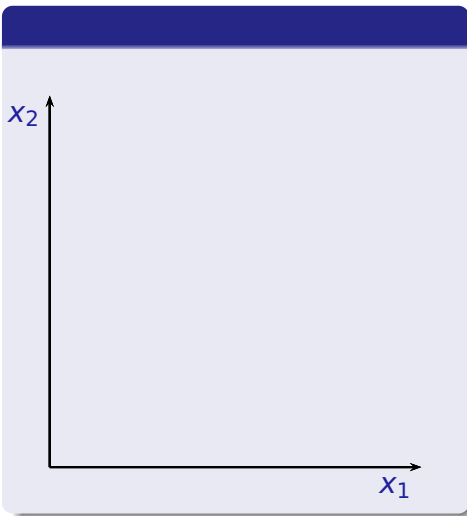
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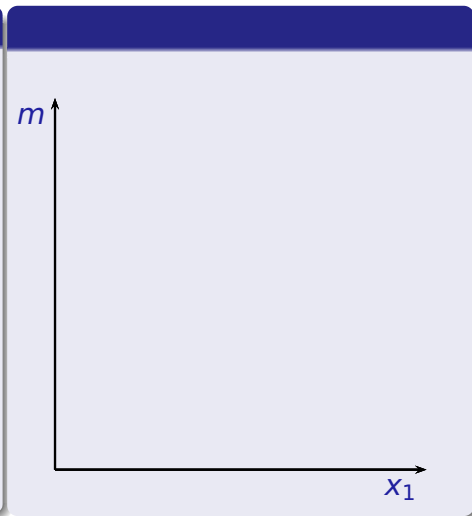
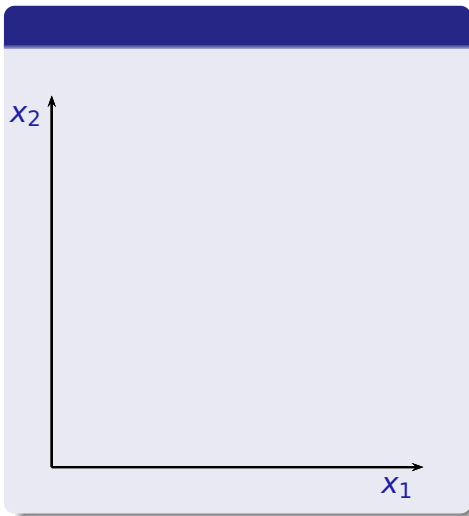


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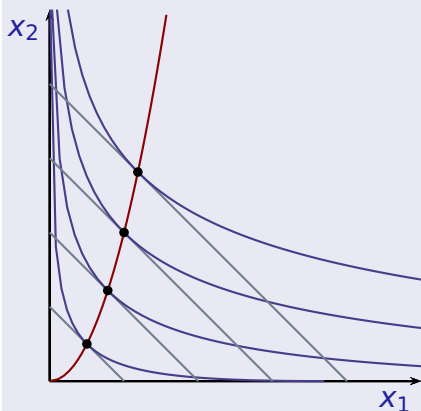


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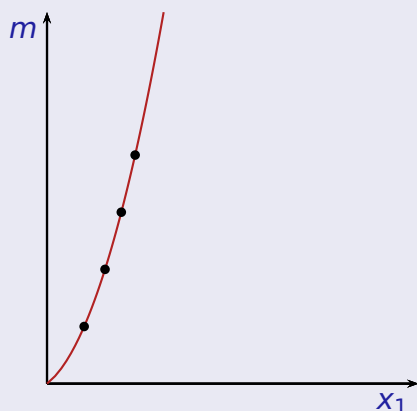


# Curva de renda consumo e curva de Engel

## Curva renda consumo



## Curva de Engel



# Elasticidade renda da demanda – definições

## Elasticidade renda da demanda no ponto

$$\epsilon_{i,m} = \frac{\partial x_i(p_1, p_2, m)}{\partial m} \frac{m}{x_i(p_1, p_2, m)}$$

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## Elasticidade renda da demanda no arco

$$\epsilon_{i,m}(p_1, p_2, m) = \frac{x_i(p_1, p_2, m + \Delta m) - x_i(p_1, p_2, m)}{\Delta m} \frac{\bar{m}}{\bar{x}_i}$$

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Na qual

$$\bar{x} = \frac{x_i(p_1, p_2, m + \Delta m) + x_i(p_1, p_2, m)}{2} \text{ e } \bar{m} = m + \frac{\Delta m}{2}$$

## Demanda Cobb-Douglas

$$x_1 = \frac{a}{a+b} \frac{m}{p_1}$$

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$$x_j = \alpha m^k$$

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$$U(x_1, x_2) = \ln x_1 + x_2$$

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Condição de 1ª ordem

$$\frac{1}{x_1} = \frac{p_1}{p_2} \Rightarrow x_1 = \frac{p_2}{p_1}$$

$$\epsilon_{im} = 0 \frac{m}{p_2/p_1} = 0$$

## Elasticidade no ponto

$$\frac{\partial p_i x_i / m}{\partial m} = \frac{p_i x_i}{m^2} (\epsilon_{im} - 1)$$

Participação do bem 1 no total de gastos aumenta, não se altera ou diminui conforme  $\epsilon_{1m}$  seja maior, igual ou menor do que 1, respectivamente.



## Elasticidade no ponto

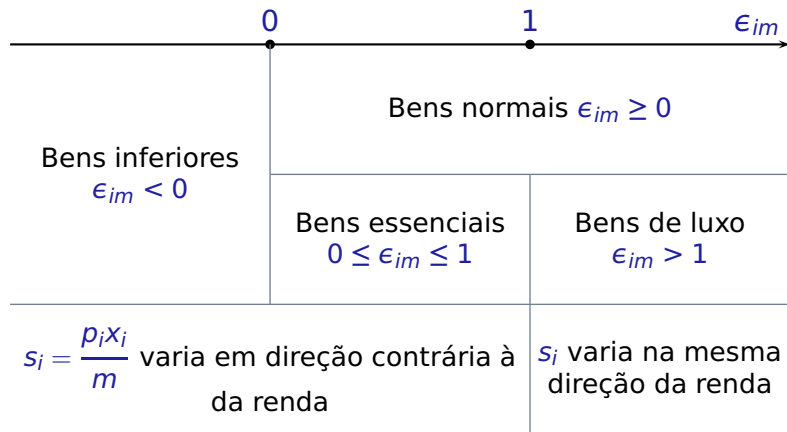
$$\frac{\partial p_i x_i / m}{\partial m} = \frac{p_i x_i}{m^2} (\epsilon_{im} - 1)$$

Participação do bem 1 no total de gastos aumenta, não se altera ou diminui conforme  $\epsilon_{1m}$  seja maior, igual ou menor do que 1, respectivamente.

## Elasticidade no arco

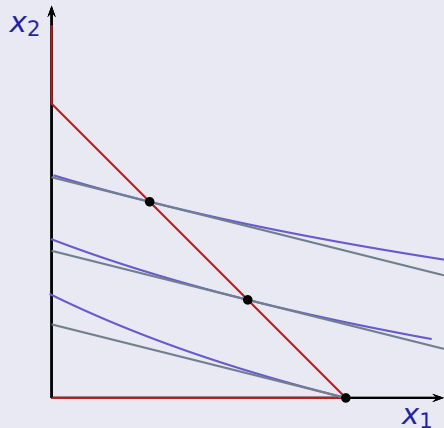
$$\frac{\Delta \frac{p_i x_i}{m}}{\Delta m} = \frac{p_i \bar{x}_i}{\bar{m}^2 - \frac{\Delta m^2}{4}} (\epsilon_{im} - 1)$$

# Classificação da demanda de acordo com sua elasticidade renda

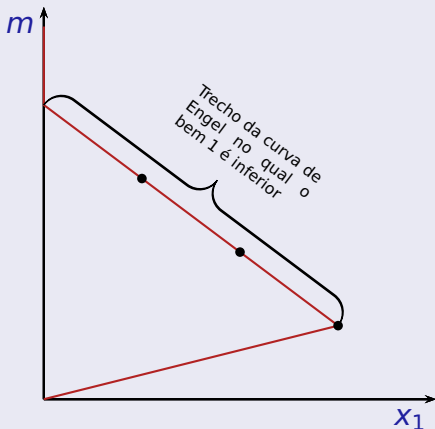


# Um bem inferior

## Renda consumo

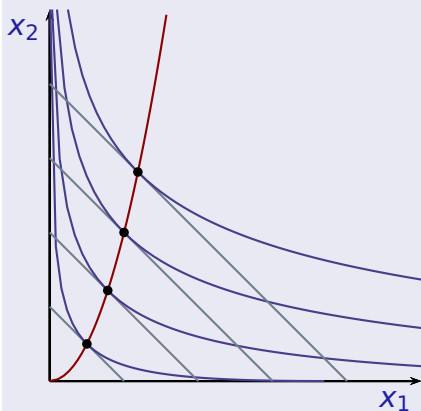


## Curva de Engel

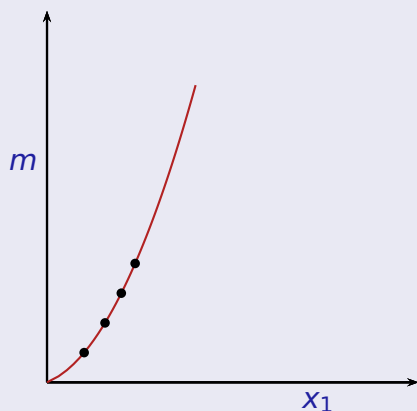


# Um bem necessário

## Renda consumo

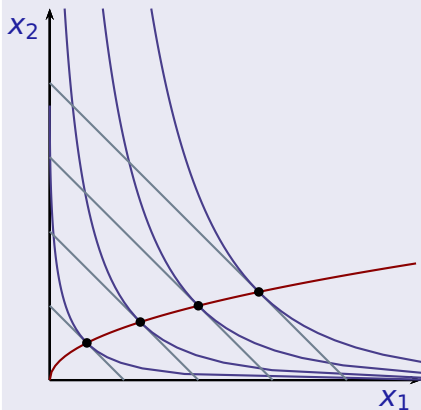


## Curva de Engel

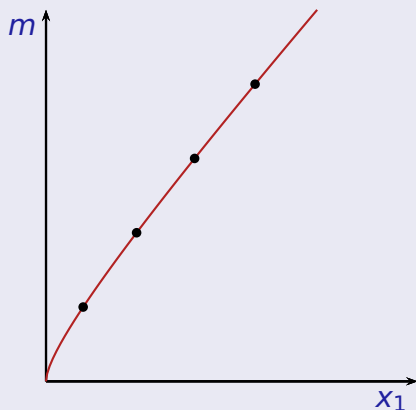


# Um bem de luxo

## Renda consumo



## Curva de Engel



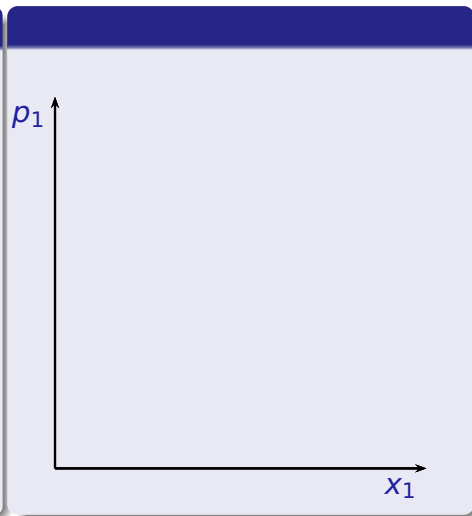
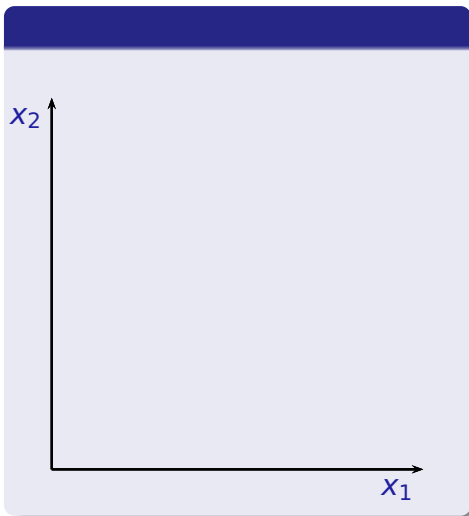
## 1 Demanda e Renda

- Curvas de renda-consumo e de Engel
- Elasticidade Renda
- Ilustrações

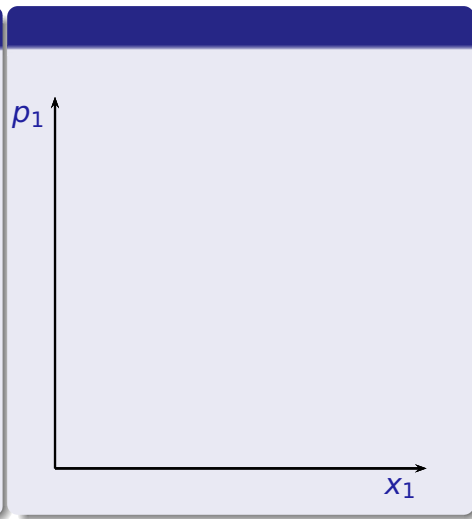
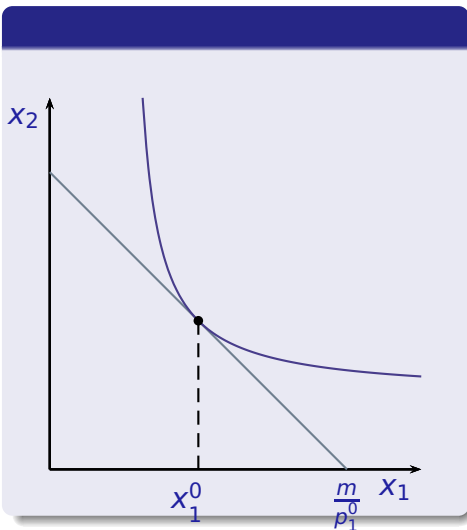
## 2 Demanda e Preço

- Curvas preço-consumo e curva de demanda
- Elasticidade preço
- Bens de Giffen
- Elasticidade preço cruzada
- Relações entre as elasticidades

# Curvas preço-consumo e curva de demanda

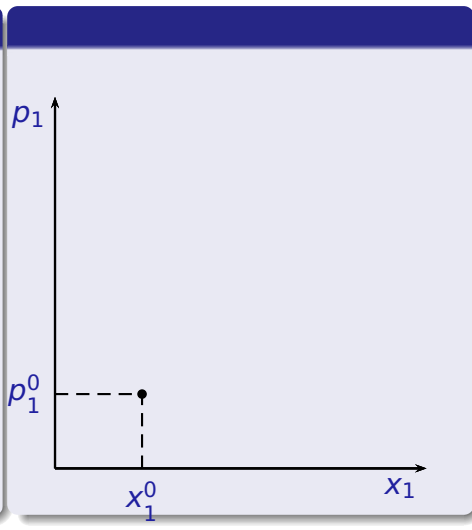
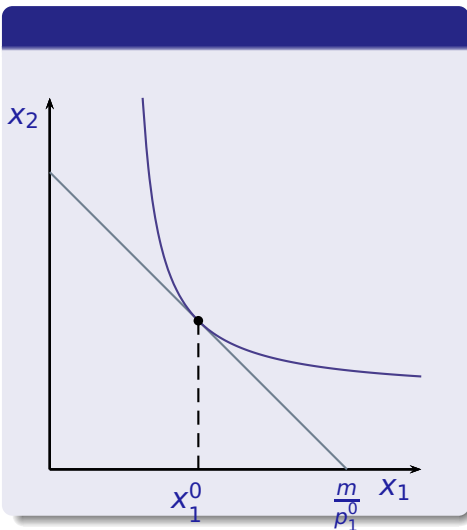


# Curvas preço-consumo e curva de demanda

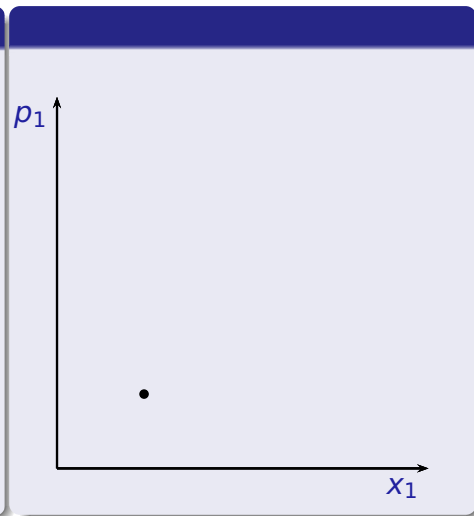
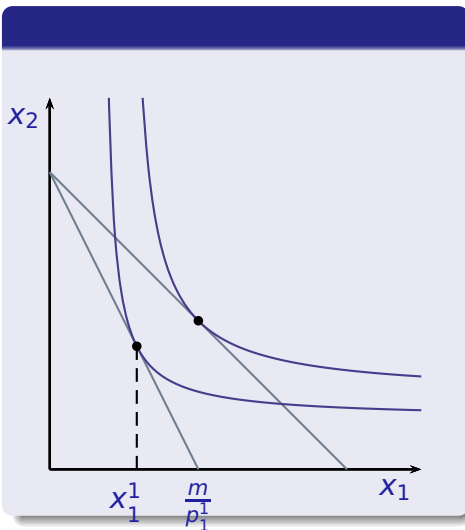




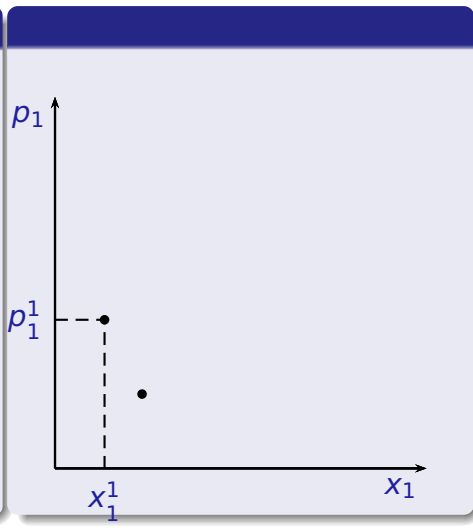
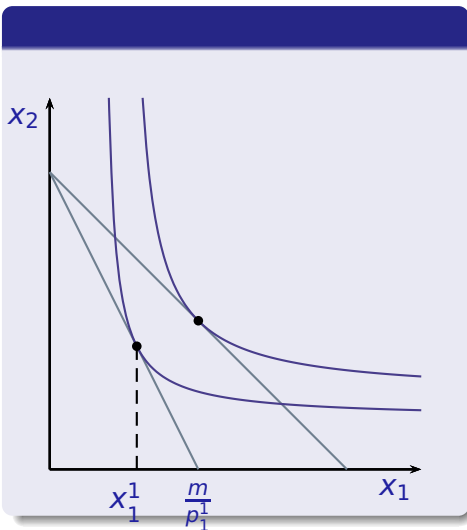
# Curvas preço-consumo e curva de demanda



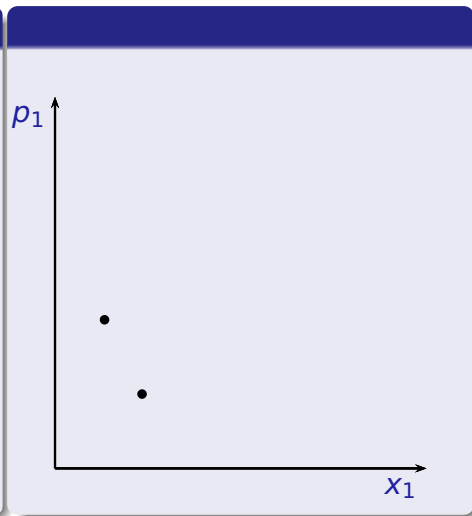
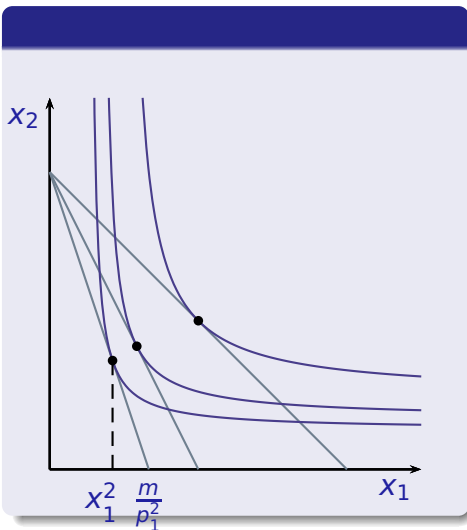
# Curvas preço-consumo e curva de demanda



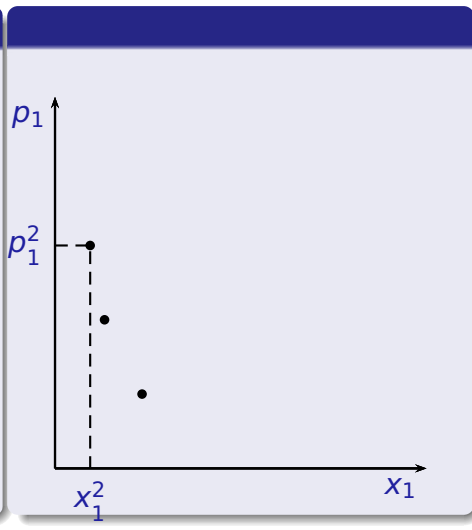
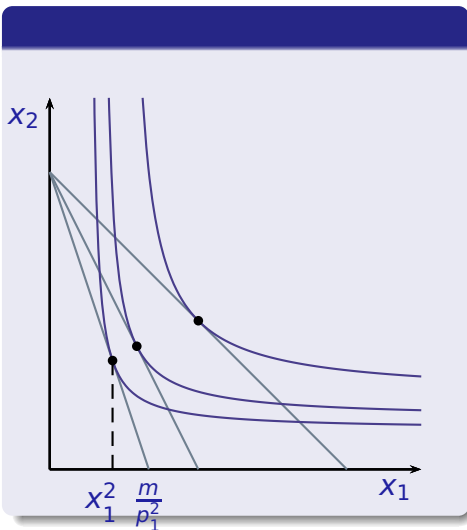
# Curvas preço-consumo e curva de demanda



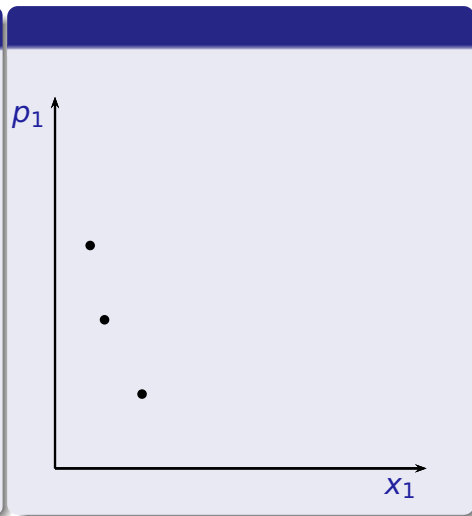
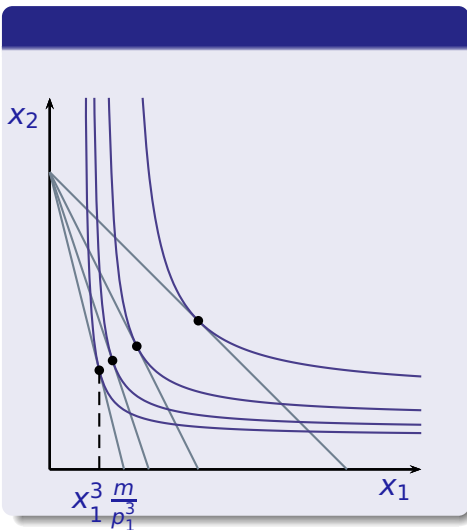
# Curvas preço-consumo e curva de demanda



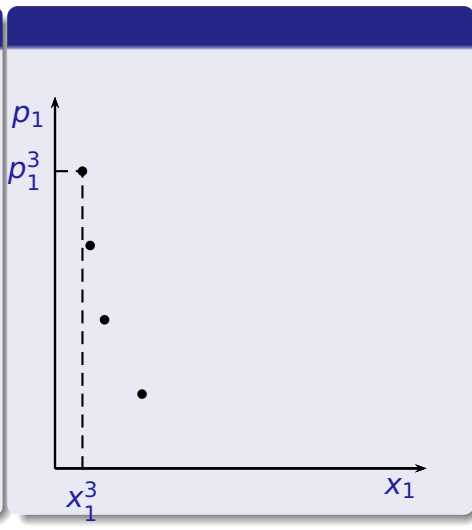
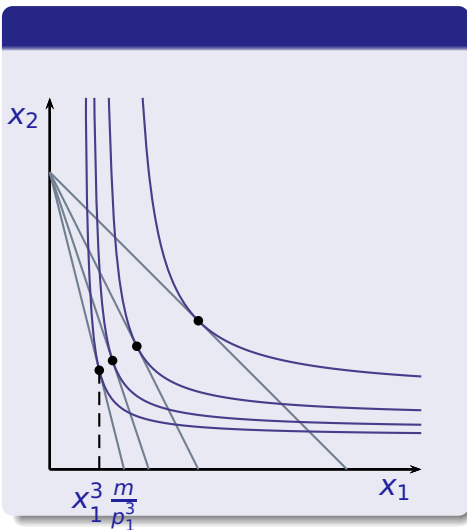
# Curvas preço-consumo e curva de demanda



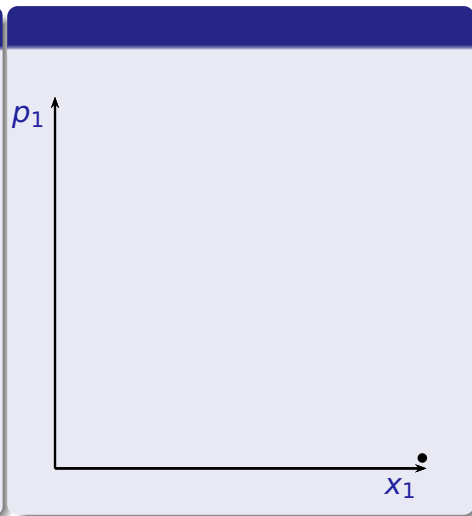
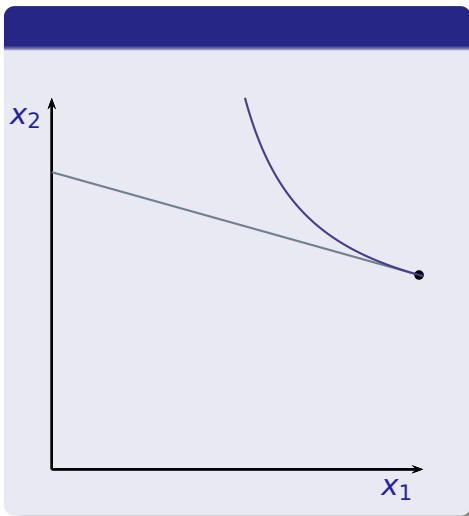
# Curvas preço-consumo e curva de demanda



# Curvas preço-consumo e curva de demanda

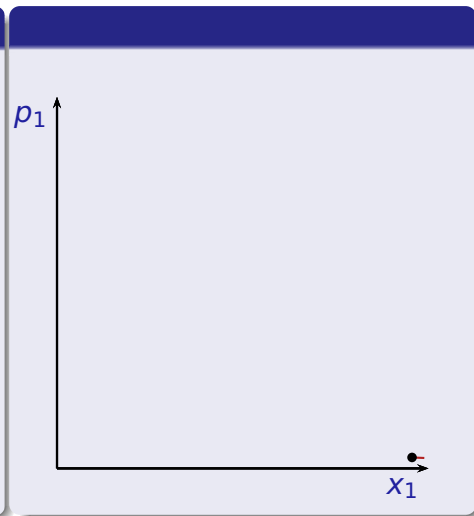
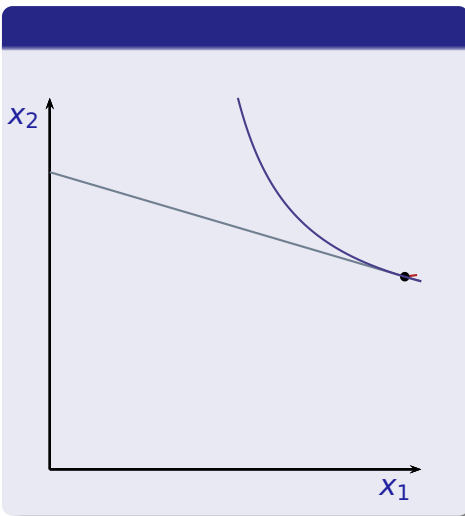


# Curvas preço-consumo e curva de demanda

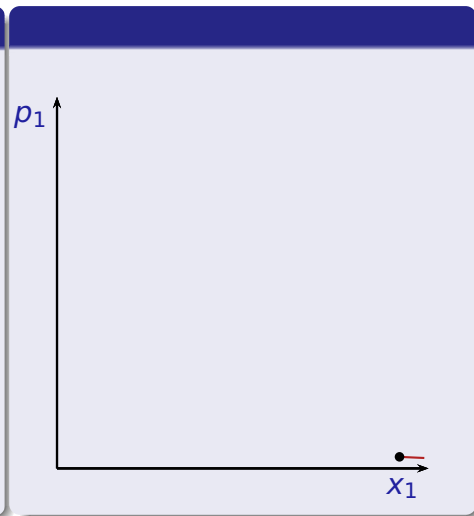
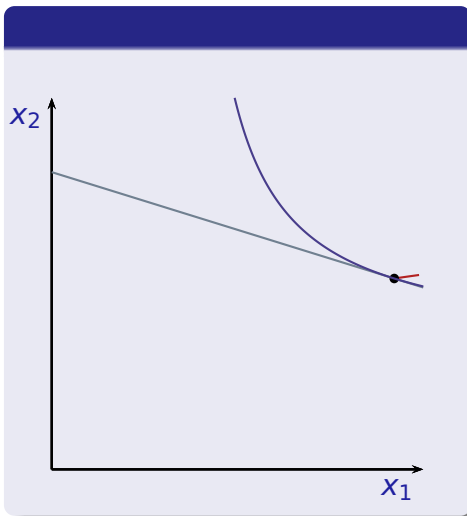




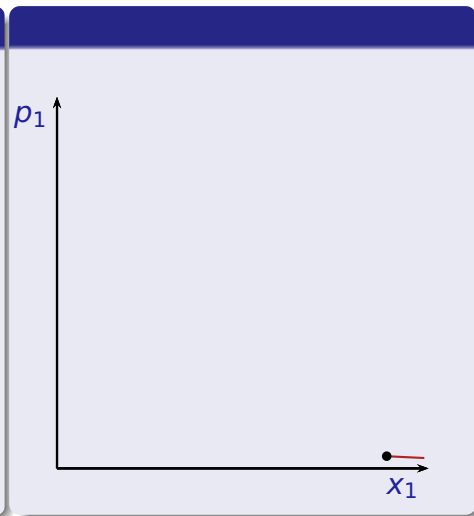
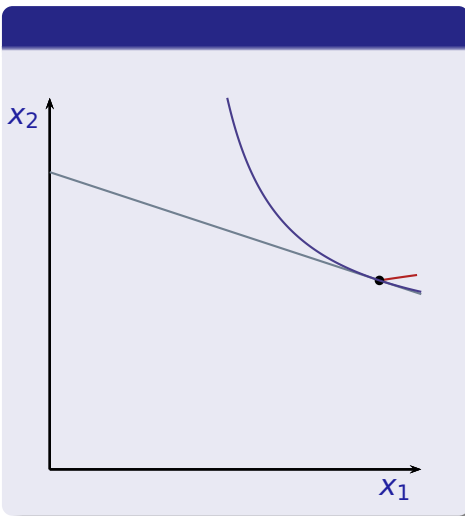
# Curvas preço-consumo e curva de demanda



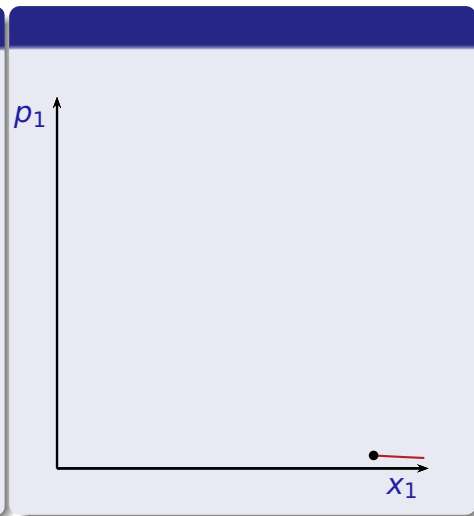
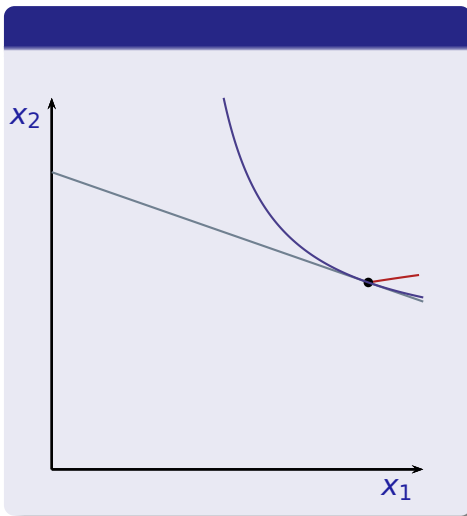
# Curvas preço-consumo e curva de demanda



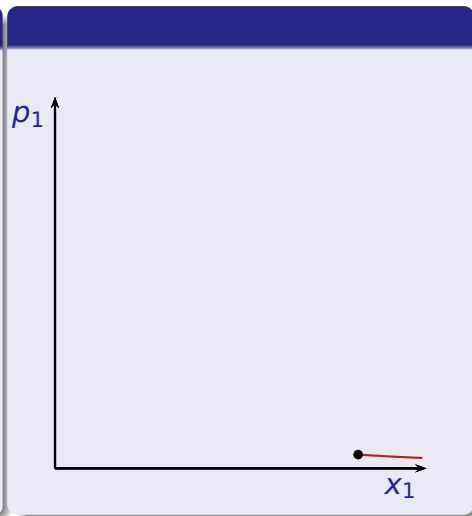
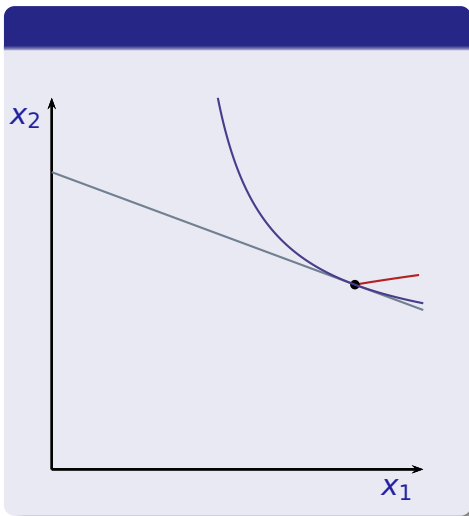
# Curvas preço-consumo e curva de demanda



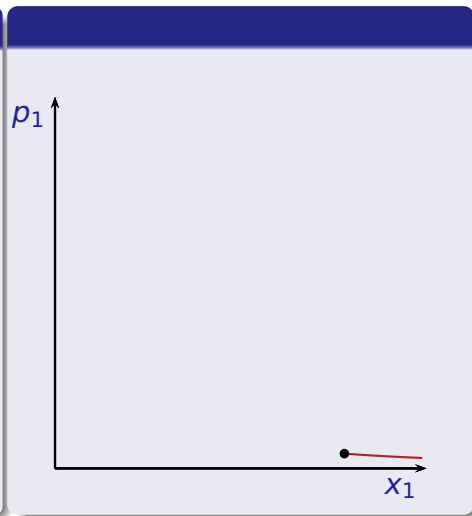
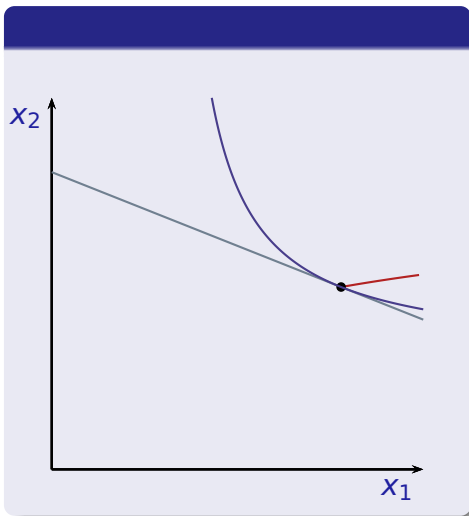
# Curvas preço-consumo e curva de demanda



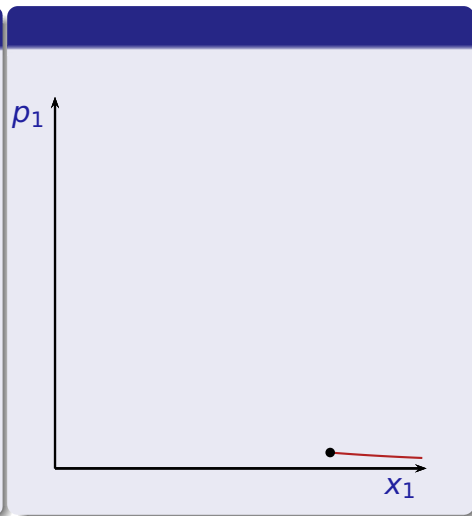
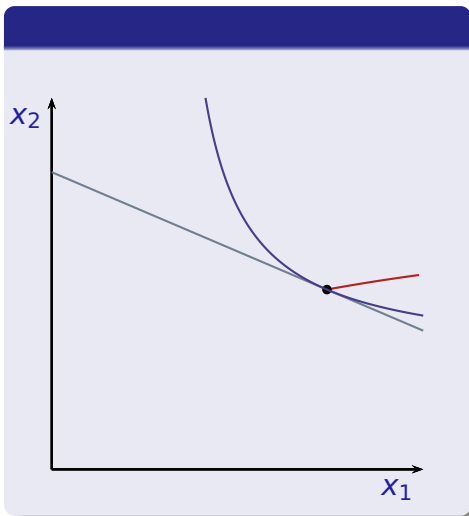
# Curvas preço-consumo e curva de demanda



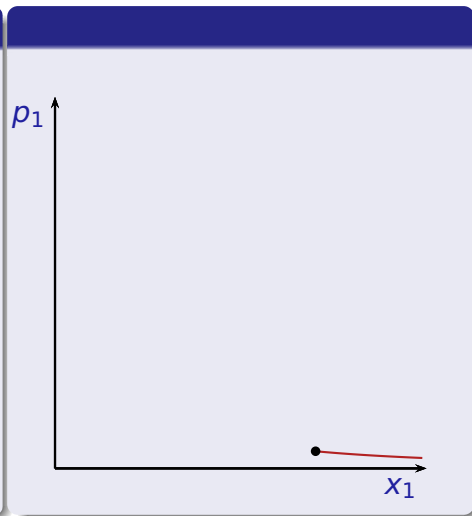
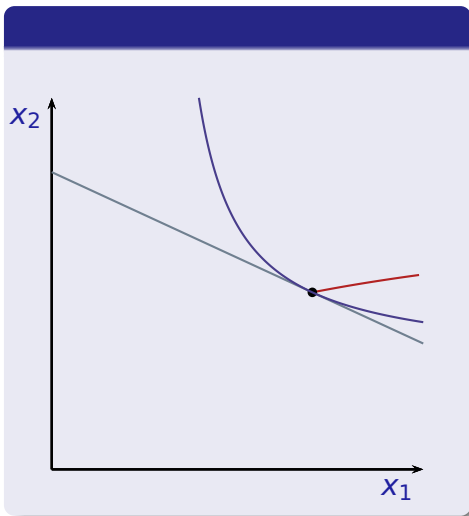
# Curvas preço-consumo e curva de demanda



# Curvas preço-consumo e curva de demanda

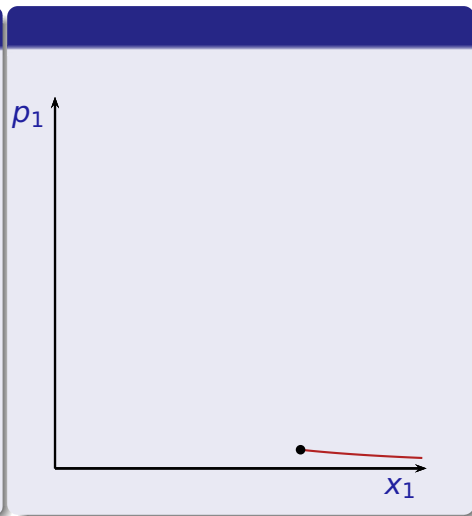
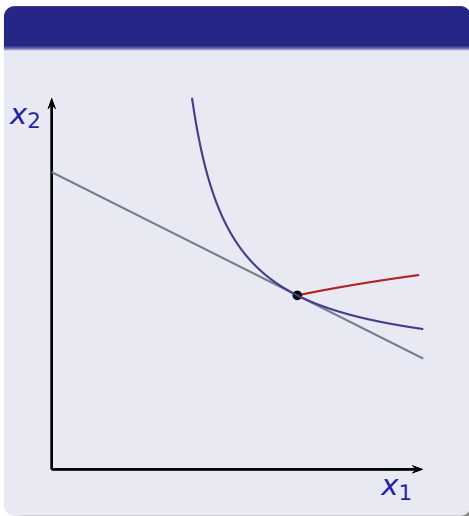


# Curvas preço-consumo e curva de demanda

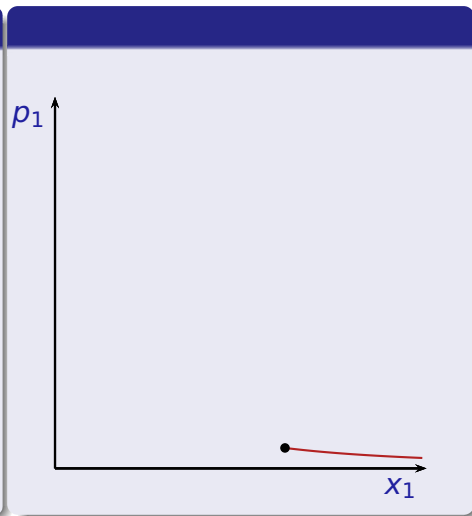
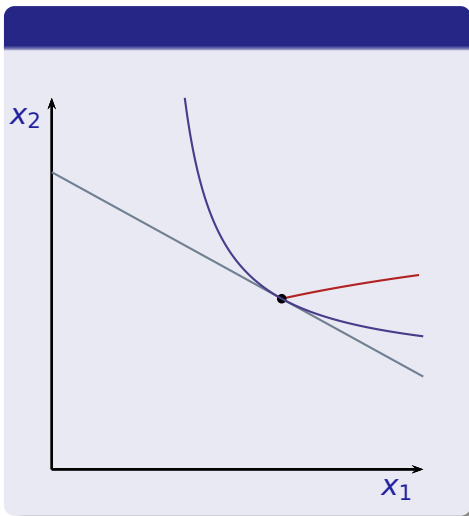




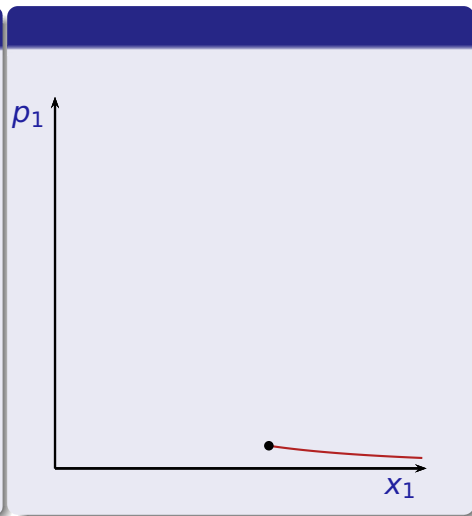
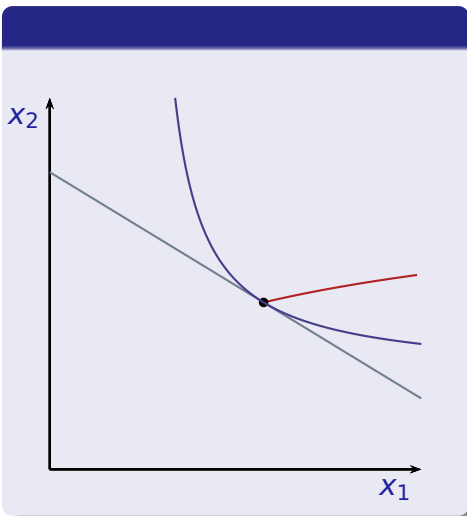
# Curvas preço-consumo e curva de demanda



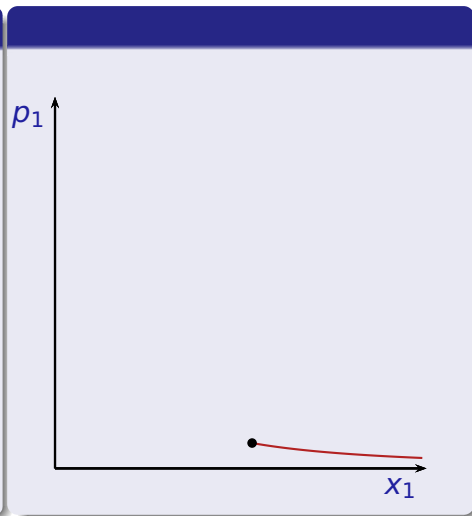
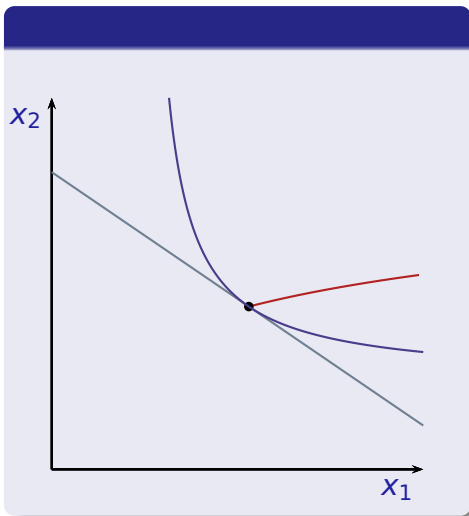
# Curvas preço-consumo e curva de demanda



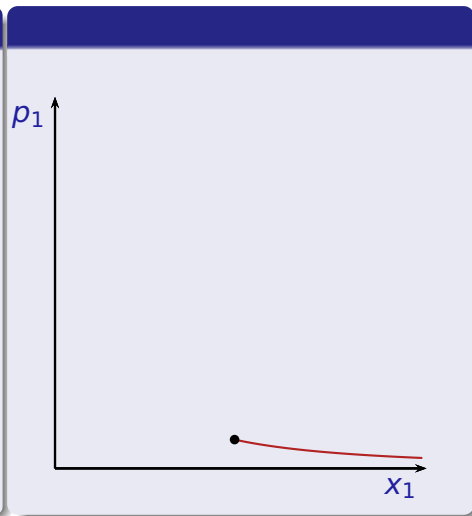
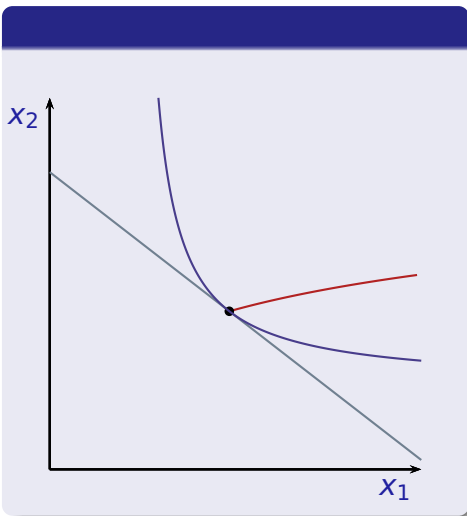
# Curvas preço-consumo e curva de demanda



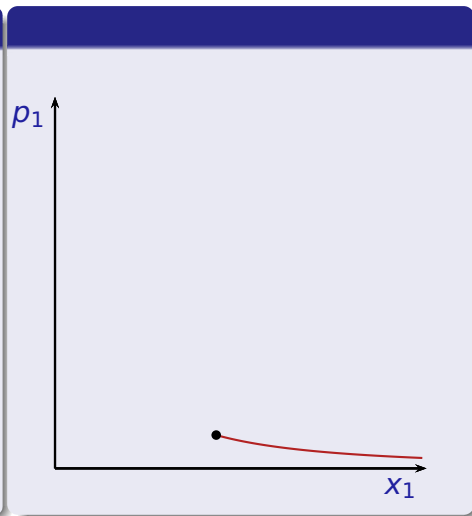
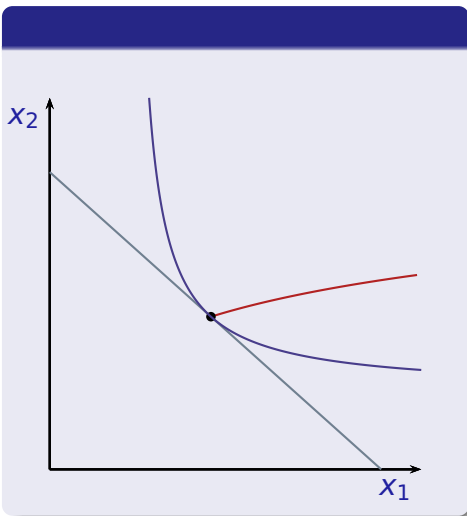
# Curvas preço-consumo e curva de demanda



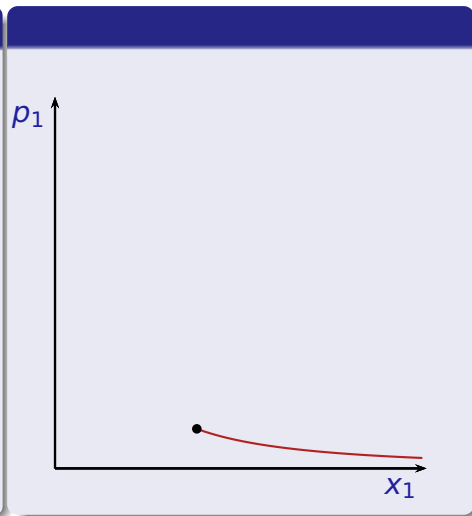
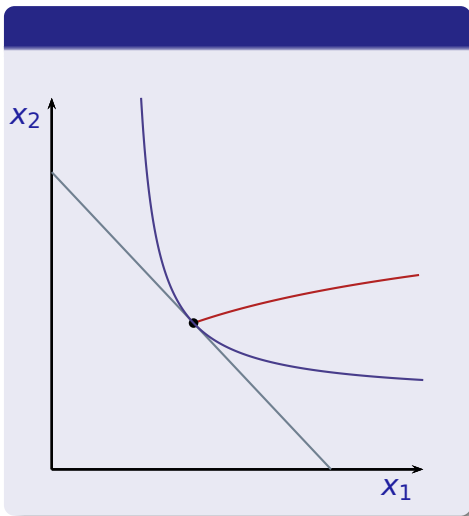
# Curvas preço-consumo e curva de demanda



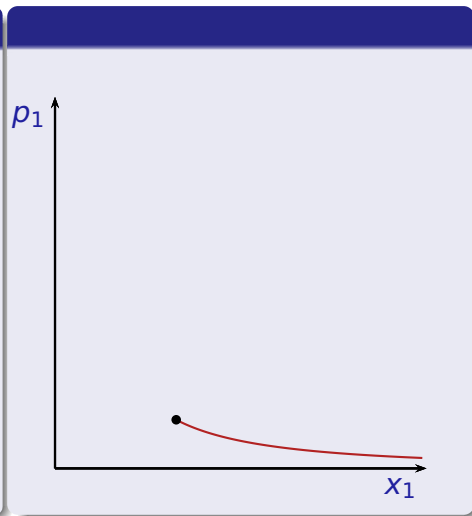
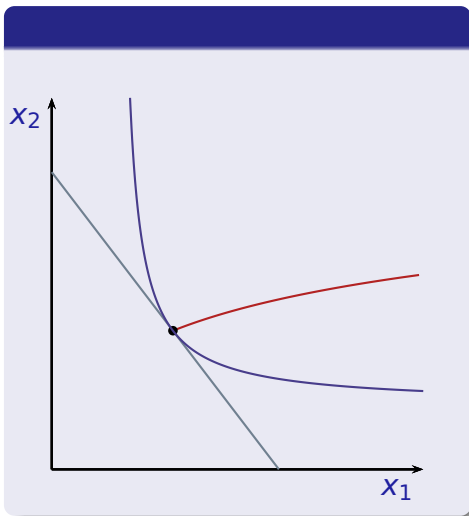
# Curvas preço-consumo e curva de demanda



# Curvas preço-consumo e curva de demanda

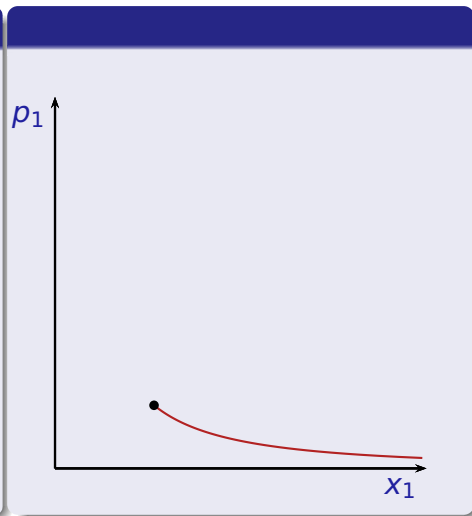
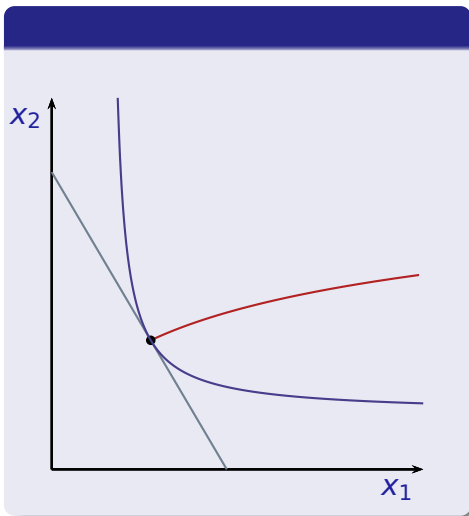


# Curvas preço-consumo e curva de demanda

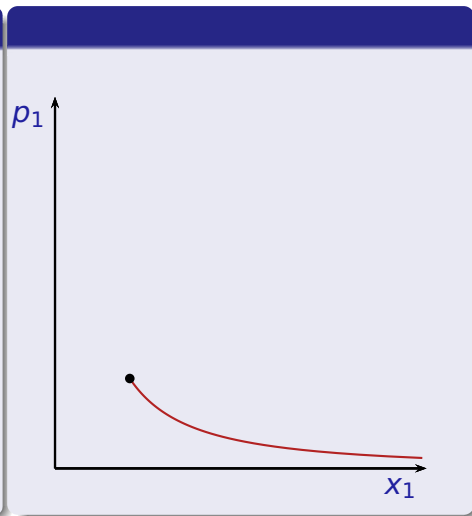
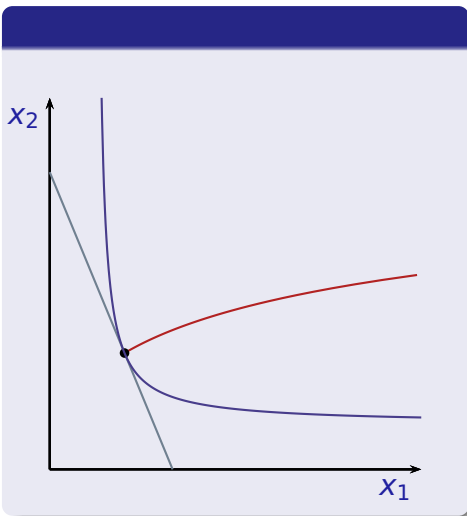




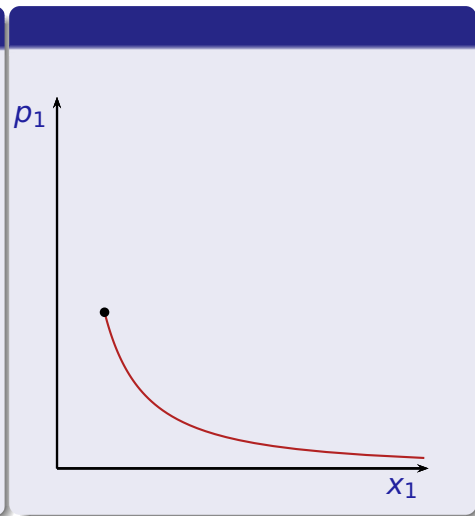
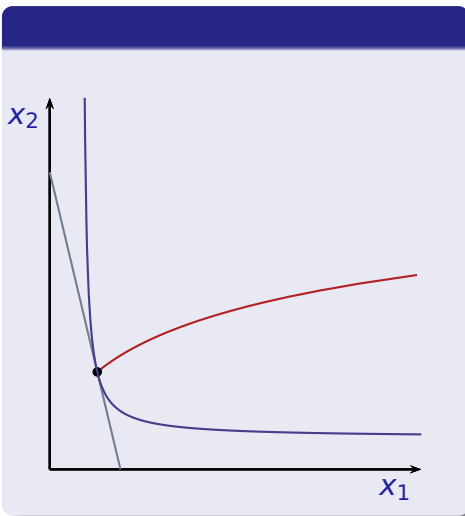
# Curvas preço-consumo e curva de demanda



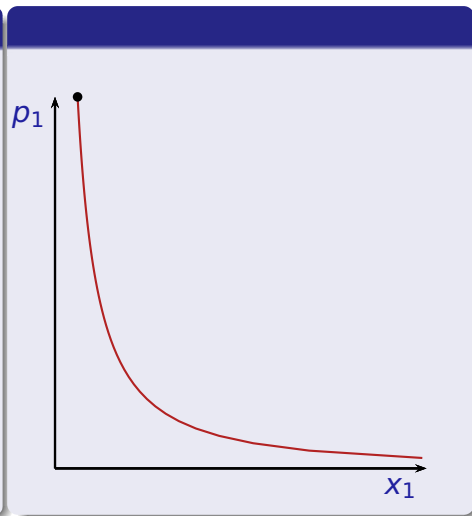
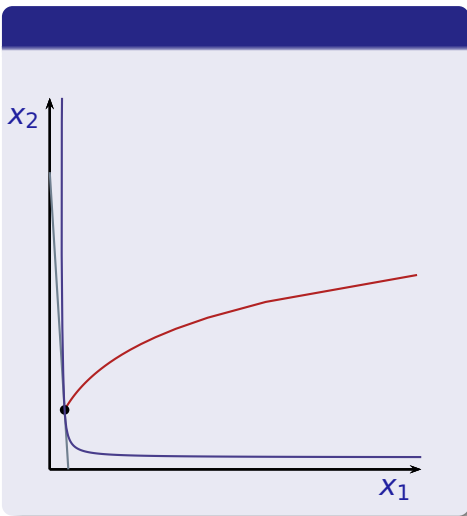
# Curvas preço-consumo e curva de demanda



# Curvas preço-consumo e curva de demanda

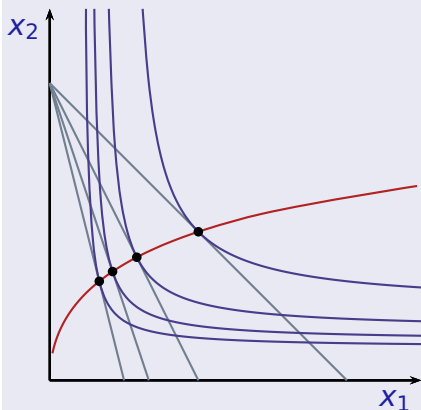


# Curvas preço-consumo e curva de demanda

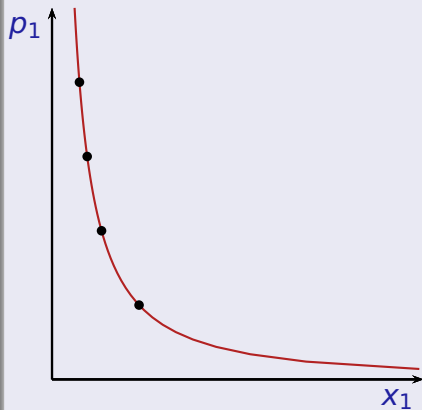


# Curvas preço-consumo e curva de demanda

## Curva preço consumo



## Curva de demanda



## Elasticidade preço da demanda no ponto

$$\epsilon_i = \frac{\partial x_i(p_1, p_2, m)}{\partial p_i} \frac{p_i}{x_i(p_1, p_2, m)} \quad i = 1, 2$$

## Elasticidade preço da demanda no ponto

$$\epsilon_i = \frac{\partial x_i(p_1, p_2, m)}{\partial p_i} \frac{p_i}{x_i(p_1, p_2, m)} \quad i = 1, 2$$

## Elasticidade preço da demanda no arco

$$\epsilon_1(p_1, p_2, m) = \frac{x_1(p_1 + \Delta p_1, p_2, m) - x_1(p_1, p_2, m)}{\Delta p_1} \frac{\bar{p}_1}{\bar{x}_1}$$

## Elasticidade preço da demanda no ponto

$$\frac{\partial p_i x_i(p_1, p_2, m)}{\partial p_i} = x_i(p_1, p_2, m)(1 + \epsilon_i)$$



## Elasticidade preço da demanda no ponto

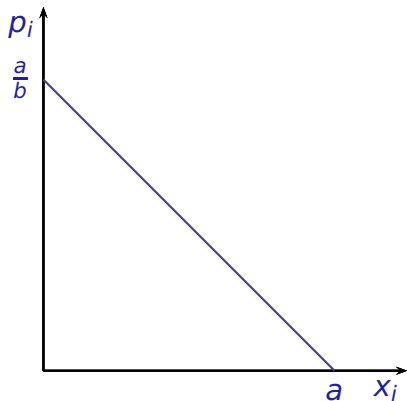
$$\frac{\partial p_i x_i(p_1, p_2, m)}{\partial p_i} = x_i(p_1, p_2, m)(1 + \epsilon_i)$$

## Elasticidade preço da demanda no arco

$$\frac{\Delta(p_i x_i)}{\Delta p_i} = \bar{x}_i(1 + \epsilon_i)$$

# Exemplo: demanda linear

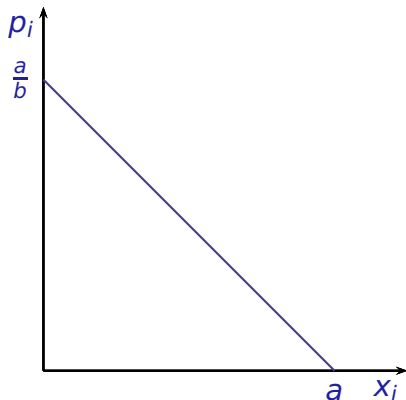
$$x_i(p_i) = a - bp_i; \quad a, b > 0$$



# Exemplo: demanda linear

$$x_i(p_i) = a - bp_i; \quad a, b > 0$$

$$\epsilon_i = -b \frac{p_i}{a - bp_i}$$

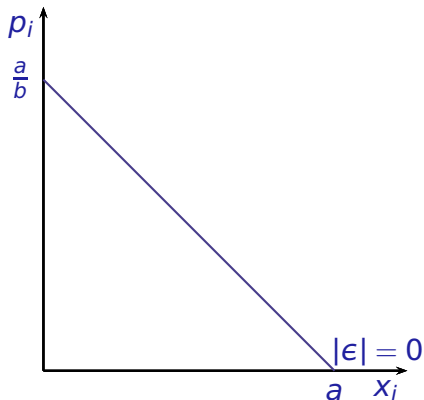


# Exemplo: demanda linear

$$x_i(p_i) = a - bp_i; \quad a, b > 0$$

$$\epsilon_i = -b \frac{p_i}{a - bp_i}$$

$$p_i = 0 \quad \Rightarrow |\epsilon_i| = 0$$



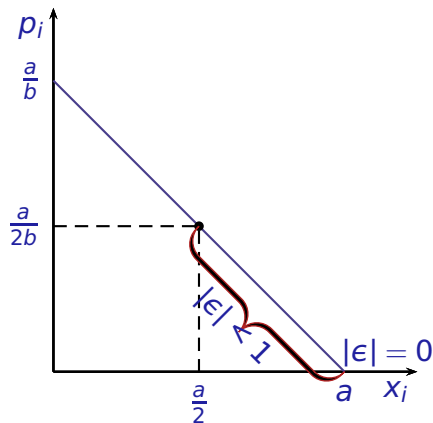
# Exemplo: demanda linear

$$x_i(p_i) = a - bp_i; \quad a, b > 0$$

$$\epsilon_i = -b \frac{p_i}{a - bp_i}$$

$$p_i = 0 \quad \Rightarrow |\epsilon_i| = 0$$

$$0 < p_i < \frac{a}{2b} \quad \Rightarrow |\epsilon_i| < 1$$



# Exemplo: demanda linear

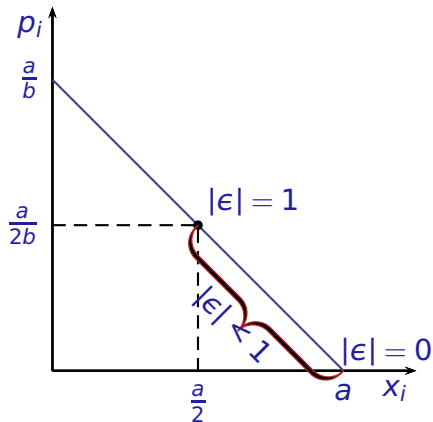
$$x_i(p_i) = a - bp_i; \quad a, b > 0$$

$$\epsilon_i = -b \frac{p_i}{a - bp_i}$$

$$p_i = 0 \quad \Rightarrow |\epsilon_i| = 0$$

$$0 < p_i < \frac{a}{2b} \quad \Rightarrow |\epsilon_i| < 1$$

$$p_i = \frac{a}{2b} \quad \Rightarrow |\epsilon_i| = 1$$



# Exemplo: demanda linear

$$x_i(p_i) = a - bp_i; \quad a, b > 0$$

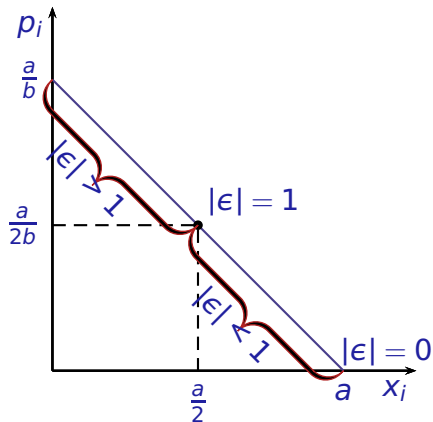
$$\epsilon_i = -b \frac{p_i}{a - bp_i}$$

$$p_i = 0 \quad \Rightarrow |\epsilon_i| = 0$$

$$0 < p_i < \frac{a}{2b} \quad \Rightarrow |\epsilon_i| < 1$$

$$p_i = \frac{a}{2b} \quad \Rightarrow |\epsilon_i| = 1$$

$$\frac{a}{2b} < p_i < \frac{a}{b} \quad \Rightarrow |\epsilon_i| > 1$$



# Exemplo: demanda linear

$$x_i(p_i) = a - bp_i; \quad a, b > 0$$

$$\epsilon_i = -b \frac{p_i}{a - bp_i}$$

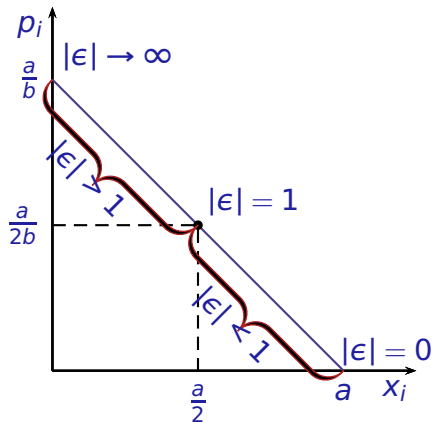
$$p_i = 0 \quad \Rightarrow |\epsilon_i| = 0$$

$$0 < p_i < \frac{a}{2b} \quad \Rightarrow |\epsilon_i| < 1$$

$$p_i = \frac{a}{2b} \quad \Rightarrow |\epsilon_i| = 1$$

$$\frac{a}{2b} < p_i < \frac{a}{b} \quad \Rightarrow |\epsilon_i| > 1$$

$$\lim_{p \rightarrow a/b^+} |\epsilon_i| = \infty$$





# Mais dois exemplos

## Demanda Cobb-Douglas

$$x_1(p_1, p_2, m) = \frac{a}{a+b} \frac{m}{p_1}$$

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## Elasticidade preço constante

$$x_i(p_i) = \alpha p_i^\epsilon$$

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# Mais dois exemplos

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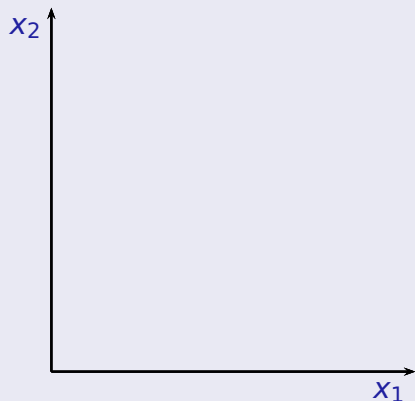
# Classificação da demanda de um bem de acordo com sua elasticidade preço

A horizontal number line representing price elasticity of demand ( $\epsilon_j$ ). The line has an arrow pointing to the right. Two points are marked on the line:  $-1$  and  $0$ . A vertical line is drawn from  $0$  down to the table below. The region to the left of  $0$  is labeled "Bem ordinário  $\epsilon_j < 0$ ". The region to the right of  $0$  is labeled "Bens de Giffen  $\epsilon_j > 0$ ".

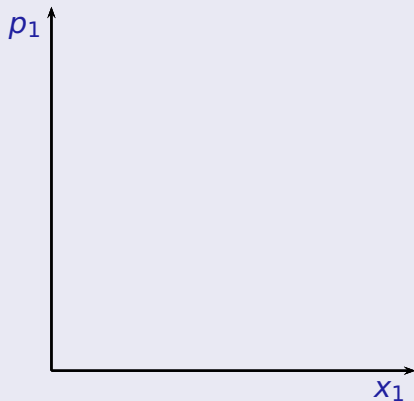
Bem ordinário $\epsilon_j < 0$		Bens de Giffen $\epsilon_j > 0$
Demanda elástica $\epsilon_j < -1$	Demanda inelástica $-1 \leq \epsilon_j \leq 0$	
$p_i x_i$ varia em direção contrária a $p_i$	$p_i x_i$ varia na mesma direção de $p_i$	

# Bens de Giffen

## Curva preço consumo



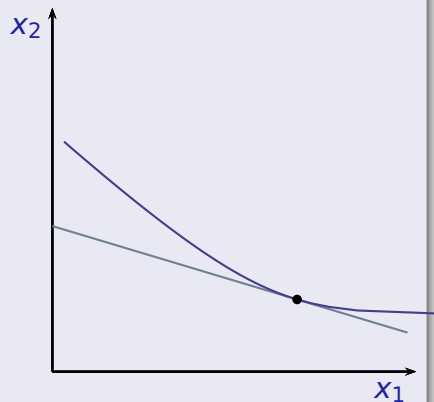
## Curva de demanda



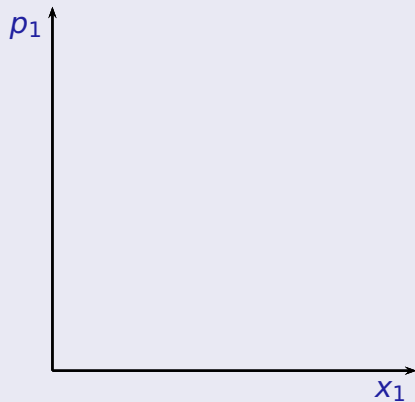


# Bens de Giffen

## Curva preço consumo

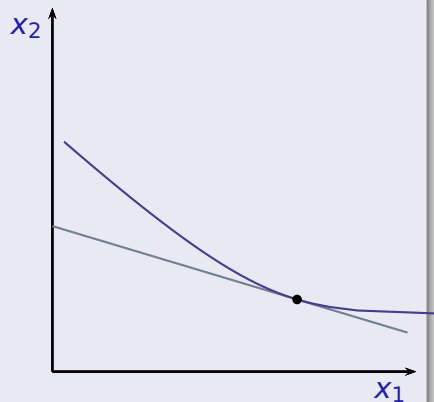


## Curva de demanda

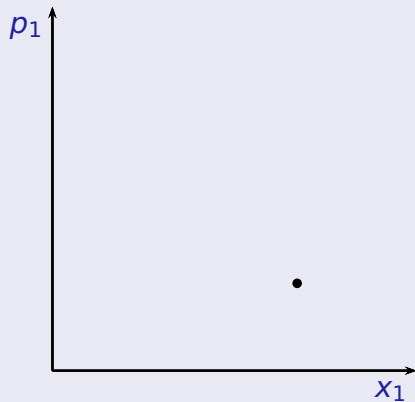


# Bens de Giffen

## Curva preço consumo



## Curva de demanda

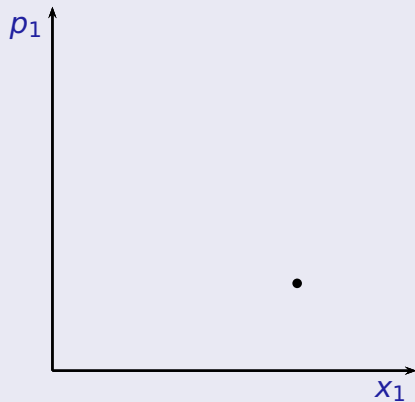


# Bens de Giffen

## Curva preço consumo

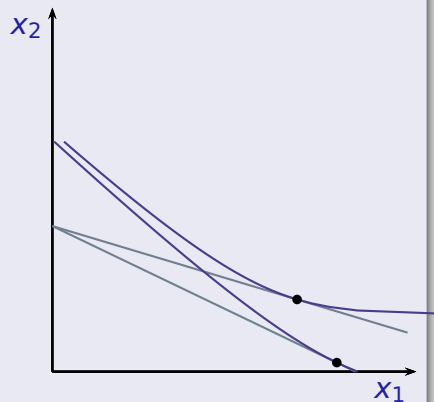


## Curva de demanda

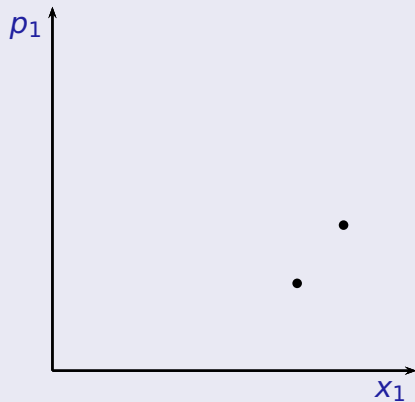


# Bens de Giffen

## Curva preço consumo

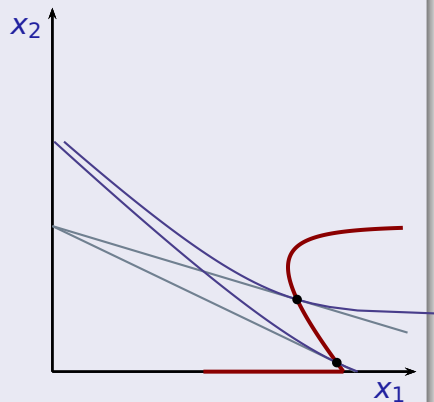


## Curva de demanda

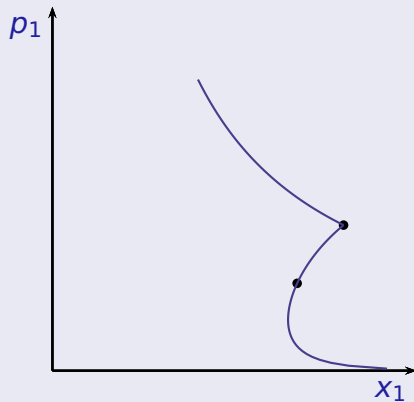


# Bens de Giffen

## Curva preço consumo



## Curva de demanda



## Elasticidade preço cruzada no ponto

$$\epsilon_{1,2} = \frac{\partial x_1(p_1, p_2, m)}{\partial p_2} \frac{p_2}{x_1(p_1, p_2, m)}$$

## Elasticidade preço cruzada no ponto

$$\epsilon_{1,2} = \frac{\partial x_1(p_1, p_2, m)}{\partial p_2} \frac{p_2}{x_1(p_1, p_2, m)}$$

## Elasticidade preço cruzada no arco

$$\epsilon_{1,2} = \frac{x_1(p_1, p_2 + \Delta p_2, m) - x_1(p_1, p_2, m)}{\Delta p_2} \frac{\bar{p}_2}{\bar{x}_1}$$

# Classificação dos bens de acordo com a elasticidade preço cruzada

$$\epsilon_{ij} > 0$$

Bem  $i$  é substituto do bem  $j$



# Classificação dos bens de acordo com a elasticidade preço cruzada

$\epsilon_{ij} > 0$	Bem $i$ é substituto do bem $j$
$\epsilon_{ij} = 0$	Bens $i$ e $j$ são independentes

# Classificação dos bens de acordo com a elasticidade preço cruzada

$\epsilon_{ij} > 0$	Bem $i$ é substituto do bem $j$
$\epsilon_{ij} = 0$	Bens $i$ e $j$ são independentes
$\epsilon_{ij} < 0$	Bem $i$ é complemento do bem $j$

## Bens independentes

$$x_1(p_1, p_2, m) = \frac{a}{a+b} \frac{m}{p_1} \quad \epsilon_{1,2} = 0$$

# Exemplos

## Bens independentes

$$x_1(p_1, p_2, m) = \frac{a}{a+b} \frac{m}{p_1} \quad \epsilon_{1,2} = 0$$

## Bens substitutos

$$x_1(p_1, p_2, m) = \frac{p_2 m}{p_1^2 + p_1 p_2} \quad \epsilon_{1,2} = \frac{2p_1 + p_2}{p_2 + p_1}$$

# Exemplos

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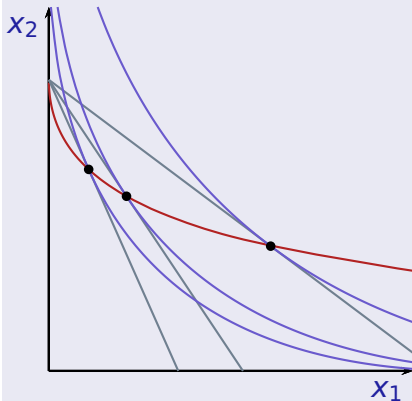
$$x_1(p_1, p_2, m) = \frac{p_2 m}{p_1^2 + p_1 p_2} \quad \epsilon_{1,2} = \frac{2p_1 + p_2}{p_2 + p_1}$$

## Bens complementares

$$x_1(p_1, p_2, m) = \frac{m}{p_1 + \sqrt{p_1 p_2}} \quad \epsilon_{12} = -\frac{2\sqrt{p_1} + \sqrt{p_2}}{2(\sqrt{p_1} + \sqrt{p_2})}$$

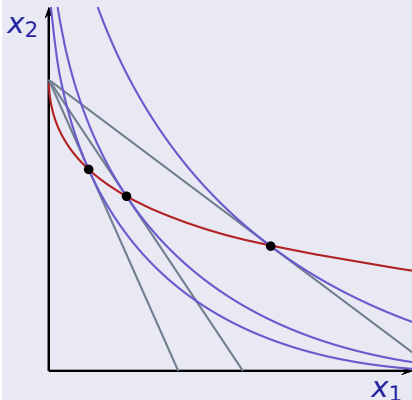
# Bens substitutos e bens complementares

## Bens substitutos

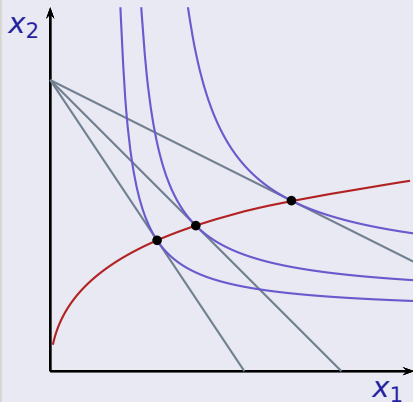


# Bens substitutos e bens complementares

## Bens substitutos



## Bens complementares



# Elasticidades e homogeneidade de grau zero

Para quaisquer  $\alpha > 0$ ,  $p_1^0, p_2^0, m^0 \geq 0$ , temos

$$x_1(\alpha p_1^0, \alpha p_2^0, \alpha m^0) = x_1(p_1^0, p_2^0, m^0).$$



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Diferenciando com relação a  $\alpha$  obtemos

$$p_1 \frac{\partial x_1(\alpha p_1^0, \alpha p_2^0, \alpha m^0)}{\partial p_1} + p_2 \frac{\partial x_1(\alpha p_1^0, \alpha p_2^0, \alpha m^0)}{\partial p_2} + m \frac{\partial x_1(\alpha p_1^0, \alpha p_2^0, \alpha m^0)}{\partial m} = 0$$

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Calculando essa igualdade para  $\alpha = 1$  e dividindo os dois lados por  $x_1(p_1^0, p_2^0, m^0)$  obtemos

$$\epsilon_{1,1} + \epsilon_{1,2} + \epsilon_{1,m} = 0$$

# Agregação de Engel

Assumindo qualquer hipótese de não saciedade local, temos, para quaisquer valores positivos de  $p_1$ ,  $p_2$  e  $m$ ,

$$p_1 x_1(p_1, p_2, m) + p_2 x_2(p_1, p_2, m) = m$$

Diferenciando essa igualdade em relação a  $m$ , obtemos

$$p_1 \frac{\partial x_1}{\partial m} + p_2 \frac{\partial x_2}{\partial m} = 1$$

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Diferenciando essa igualdade em relação a  $m$ , obtemos

$$p_1 \frac{\partial x_1}{\partial m} + p_2 \frac{\partial x_2}{\partial m} = 1$$
$$\frac{p_1 x_1}{m} \frac{m}{x_1} \frac{\partial x_1}{\partial m} + \frac{p_2 x_2}{m} \frac{m}{x_2} \frac{\partial x_2}{\partial m} = 1$$

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$$s_1 \epsilon_{1,m} + s_2 \epsilon_{2,m} = 1$$

# Agregação de Cournot

$$p_1 x_1(p_1, p_2, m) + p_2 x_2(p_1, p_2, m) = m$$

Diferenciando essa identidade em relação a  $p_1$ , vem

# Agregação de Cournot

$$p_1 x_1(p_1, p_2, m) + p_2 x_2(p_1, p_2, m) = m$$

Diferenciando essa identidade em relação a  $p_1$ , vem

$$x_1 + p_1 \frac{\partial x_1}{\partial p_1} + p_2 \frac{\partial x_2}{\partial p_1} = 0$$

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$$p_1 x_1(p_1, p_2, m) + p_2 x_2(p_1, p_2, m) = m$$

Diferenciando essa identidade em relação a  $p_1$ , vem

$$x_1 + p_1 \frac{\partial x_1}{\partial p_1} + p_2 \frac{\partial x_2}{\partial p_1} = 0$$
$$\frac{p_1 x_1}{m} + \frac{p_1}{m} \frac{\partial x_1}{\partial p_1} p_1 + \frac{p_2}{m} \frac{\partial x_2}{\partial p_1} p_1 = 0$$



# Agregação de Cournot

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Diferenciando essa identidade em relação a  $p_1$ , vem

$$\begin{aligned}x_1 + p_1 \frac{\partial x_1}{\partial p_1} + p_2 \frac{\partial x_2}{\partial p_1} &= 0 \\ \frac{p_1 x_1}{m} + \frac{p_1}{m} \frac{\partial x_1}{\partial p_1} p_1 + \frac{p_2}{m} \frac{\partial x_2}{\partial p_1} p_1 &= 0 \\ \frac{p_1 x_1}{m} + \frac{p_1 x_1}{m} \frac{\partial x_1}{\partial p_1} \frac{p_1}{x_1} + \frac{p_2 x_2}{m} \frac{\partial x_2}{\partial p_1} \frac{p_1}{x_2} &= 0\end{aligned}$$

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Diferenciando essa identidade em relação a  $p_1$ , vem

$$x_1 + p_1 \frac{\partial x_1}{\partial p_1} + p_2 \frac{\partial x_2}{\partial p_1} = 0$$

$$\frac{p_1 x_1}{m} + \frac{p_1}{m} \frac{\partial x_1}{\partial p_1} p_1 + \frac{p_2}{m} \frac{\partial x_2}{\partial p_1} p_1 = 0$$

$$\frac{p_1 x_1}{m} + \frac{p_1 x_1}{m} \frac{\partial x_1}{\partial p_1} \frac{p_1}{x_1} + \frac{p_2 x_2}{m} \frac{\partial x_2}{\partial p_1} \frac{p_1}{x_2} = 0$$

$$S_1 \epsilon_{1,1} + S_2 \epsilon_{2,1} = -S_1$$